A TREATISE OF PERSPECTIVE DEMONSTRATIVE AND PRACTICAL.

Illustrated with Copper CUTTS.

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To the Honourable
Francis Nicholson, Esq;
General of Her MAJESTY's
FORCES
IN
North America;
A
True Patriot, a Gentleman, and a Friend,

To whom his COUNTRY is indebted for many signal Services Abroad, as LEARNING is, for a Generous Encouragement at Home;

THIS
Treatise of PERSPECTIVE
(Design'd for the Use of the *NEW MATHEMATICAL-SCHOOL* in *Christ's-Hospital*) is humbly Dedicated,

In Testimony of
That profound RESPECT
Which is (and ever will be) paid to him

BY THE
AUTHOR.

PREFACE
TO THE
READER.

SINCE Custom has made it a Point of Civility to the Reader, as well as a sort of Ornament to a Book, to introduce it with some Prefatory Discourse; I shall endeavour to make the following one as *Useful* as I can, by discoursing of something by which I may inform the Reader, at the same time that I pay him the usual Complement.

And therefore as the Design of the Ensuing Treatise, is to explain the Nature and Properties, of *one particular Sort or Kind of* PROJECTION; so I propose here to explain, the Nature of PROJECTION IN GENERAL, with its several *Kinds*, and their *Uses* and *Differences* one from another; and this, as far as the Bounds that are here set me will permit.

PROJECTION, is the Transcription or Delineation, of an Object upon a Plane. Or rather; *'Tis the Figure, mark'd or trac'd out upon a Plane, by a moveable Line, extended from the EYE, as a common Pole or Centre, to the several Points of an Object.* Upon this Account, 'tis called by some, by the Name of SECTION, and that not improperly; for that Figure,

Image, or Repreſentation of an Object upon a Plane, which we call PROJECTION; is no other than the SECTION of the Viſual *Cone*, *Pyramid*, *Cylinder*, or *Priſm*, by the Plane on which the ſaid Figure is *deſign'd*. It's eaſily underſtood from hence, how great a *Variety of Projection* ariſes, from the *various Poſitions*, both of the *Eye*, the viſible *Object*, and the *Plane* it ſelf. That as it muſt needs be vaſtly different, if the *Eye* and *Plane* continuing their Situation, the *Object* changes from a *direct* Poſition to an *oblique*; ſo likewiſe it muſt be, if the *Eye* and *Object* remaining as they were, the *Plane* be mov'd from one Situation to another. And *that if the Plane, be between the Object and the Eye*; the Projected Figure will be *leſs*, the nearer the Plane is to the *Eye*, and the *greater*, the further off; ſuppoſing the Eye and Object to be fix'd, and the Plane to move: As alſo that the projected Figure will be *greater*, the further the *Eye* is from the *Plane*, and *leſs*, the nearer it is to it; ſuppoſing the *Object* and *Plane* to be fix'd, and the Eye to move backwards or forwards. On the other hand; *that if the Object, be between the Plane and the Eye*: Then the further the Object is from the Plane, the *bigger* its Projection is, and the nearer, the *leſs*; ſuppoſing the *Plane* and the *Eye* to retain their Poſitions, and the *Object* to move; or that the further the *Eye* is from the Plane, the *leſs* the Projection of the Object is, and the nearer, the bigger; ſuppoſing the *Object* and *Plane* to retain their Poſitions, and the *Eye* to move.

Theſe things are obvious, upon the drawing of three or four ſtrait Lines. And therefore paſſing this, I think it not improper to obſerve in the next Place, that we ought to conceive a Difference, between

tween the PROJECTION, and the bare APPEARANCE, of an Object to the Eye. For the Situation of the *Object*, and of the *Eye*, continuing, the APPEARANCE is still the same: But tho' the *Eye* and *Object* should retain their Positions, yet if the *Plane* alters its Situation, the PROJECTION will not be the same, but very different. So that these Two are not entirely the same; nor are the Words therefore to be used promiscuously, as Terms perfectly equivalent, and that signifie one and the same thing. APPEARANCE depends only upon *the Relation of two things* to each other, *viz.* the *Object*, and *Eye*: But PROJECTION besides those *Two*, takes in the Consideration of a *Plane*, which besides a vast *Variety* in it self, introduces a considerable *Difference* between it and the other. Yet after all; *Projection* is no more than *Relative Appearance*; that is, such as results, from this or that particular Situation, of *Eye*, *Object*, and *Plane*, altogether. And 'tis this particular Consideration of a *Plane* likewise, that distinguishes *this Science*, from what we commonly call, SIMPLE or DIRECT OPTICKS. For as *there*, we consider Quantities purely as VISIBLE, or as the Objects of Vision; so *here*, they are consider'd, as Visible, *with Respect to a certain Plane, lying in this or that particular Position.*

The *different Kinds or Species* of Projection, must be taken, from the various placing, either of the *Object*, the *Plane*, or the *Eye*.

That they ought not to be deriv'd from the Positions of the *Object*; is plain. Because, tho' very different Projections will arise from hence, yet they would be *Infinite*; even as many, as there are Positions, that the Object may be plac'd in.

 Besides,

Besides; from a different POSITION, arises a different APPEARANCE to the Eye: And since Projection, is only transcribing the Object as it *appears*; we should thus, rather be projecting so many *several different Objects*, than making the several different Projections of one and the same Object.

Neither ought this Distinction, to be taken from the various *Positions of the Plane*, which in general can be but Three, viz, *Perpendicular*, *Parallel*, or *Oblique*, to a Ray, let fall perpendicularly, from the Eye, to the Object.

'Tis true, that there arise from hence, three very different Projections; and a Man may, if he pleases, call them different *Kinds* of Projections too: But however, it would be of little Use or Advantage to distinguish them thus; nay it would be (without farther Limitations and Conditions) an obscure, doubtful and ambiguous Way of giving an Account of a Projection, to say, *It was such a one, that the primary visual Ray, was at Right Angles, or not at Right Angles, to the Plane, on which the Projection was made.*

A Man by this, might possibly, in many Cases, understand one Sort of Figure, when in Reality, it was quite another that was intended.]

Certainly the Distinction of the *Kinds* of Projection, ought to be taken from that Principle, and that only, which will infer the most compleat, comprehensive, and easily conceivable Differenee, between the Members so distinguish'd; and that Principle must of necessity be, *the various Distance of the Eye.*

This takes in and accounts for all; and introduces a clear and distinct Notion, of Three Kinds of Projection, vastly different from one another.

Nor

Nor can there, upon this Principle, poſſibly be any more than *Three*; ſince there can be but a threefold Variety in that Article of Diſtance. For the Eye may be ſuppos'd, either to be *Infinitely remov'd*, or *Infinitely near*, (or in *Contact*, as they expreſs it) or elſe at ſome *juſt and moderate Diſtance*.

Accordingly we have (what the Writers of this Science, have call'd by the Name, of the) ORTHOGRAPHICAL, STEREOGRAPHICAL, and SCENOGRAPHICAL, Projections: Of each of which, we ſhall ſpeak ſomething in their Order.

In the *Orthographick* Projection, we commonly ſay, the *Eye* is ſuppos'd to be at an *Infinite Diſtance*; which is not to be underſtood *ſtrictly*, but *comparatively* ſo; or in a more rude and vulgar way of Speech, for an *Immoderate or very great Diſtance*, and which with Reſpect to our *Ordinary Views* (which are taken at *ſmall Diſtances*) may well enough be call'd Infinite. We may fairly reckon that to be an *Immoderate* (and in this Senſe therefore an *Infinite*) Diſtance, when the *Parts* of an Object, which in *reality*, bear a very conſiderable Proportion to the *whole*; do notwithſtanding diſappear and lie hid, ſo that we can't diſcern *Exceſſes and Defects*, or make Compariſons between them, as we could eaſily do, at ſome other Stations leſs diſtant from that Object.

And therefore, this *Infinite Diſtance* we ſpeak of, is ſo far from conſiſting *in Indiviſibili*, or being *one* only Immenſe Diſtance; that it admits of a great Latitude; nay is capable of *Infinite Variety*, according to the *Magnitude* and *Extent* of the Objects view'd or conſider'd. That ſame Diſtance, which with Reſpect to a *very ſmall* Object, may be *Immoderate* and Exceſſive; with Reſpect to a *Great* one, may be *Juſt* and Moderate enough. Or a very ſmall

Diſtance, in Compariſon to a minute Object, may be immoderate; when a vaſtly great one may be the Contrary, with Reſpect to an Object of proportionally large Dimenſions. The *Moon's* Diſtance from the *Earth* is properly enough ſtil'd *Infinite*, in Compariſon to ſome petty Meaſures of Length and Diſtance, in common Uſe here amongſt us. But yet it is not ſo, with Reſpect to the *Semidiameter, of the Terreſtrial Globe*. For we find (for Example) that the Appearances of *Solar Eclipſes* are very different, at the very ſame Moment of *Abſolute* Time; to People that obſerve them, from different Parts of the Globe: Which ſhews, that the Semidiameter of our Earth is far enough from being as a *Point*, with Reſpect to the Diſtance of the *Eclipſing* Luminary, and does indeed bear ſome conſiderable Proportion thereto; and this Proportion is commonly expreſs'd in round Numbers, by that of 1 to 60. However this ſame Semidiameter of the *Terreſtial Globe*, bears no *ſenſible* Proportion, to the *Sun's* Diſtance from it; which therefore is in our Senſe, an *Infinite* Diſtance. Hence we take the Sun's Rayes as *Parallel*, and determine the *Foci* of them in *Refracting* or *Reflecting Glaſſes*, as for Rayes that are really Parallel; and that without conſiderable Errour. We ſuppoſe the Sun to enlighten *Half* the Globe of our Earth; when as in *Geometrical* Strictneſs, 'tis certain that he enlightens *more* than a Hemiſphere. But then, as one and the ſame Luminary, enlightning one and the ſame *Spherick* Body which is *leſs* than that Luminary; enlightens *a leſs Portion* of it (tho' always *more* than a Hemiſphere) at a *greater* Diſtance, than it does at a *leſs* Diſtance; ſo upon the Account of an Immoderately great Diſtance, between the two Bodies; the enlightned Part will approach

ſo

ſo near to a Hemiſphere, or rather, the Exceſs of the Enlightned Part above a Hemiſphere, will be ſo far diminiſh'd; that no *ſenſible* Difference will ariſe.

And this is the Caſe, with Reſpect to the *Sun*, and the Globe we live on; upon which Score (tho' it be not *Mathematically* True) we ſay, that *Half the Latter* is enlightned by the *Former.* So alſo, we take *the Shadows of Equidiſtant Gnomons, to be Parallel to one another*; and ſay, that 'tis the ſame thing, whether *Dials* are plac'd on the Surface, or at the *Centre* of the Earth; whereas rigorouſly ſpeaking, neither are, nor can, the Shadows of ſuch Gnomons be parallel (unleſs in one Caſe, when the Gnomons themſelves, are diſpos'd parallel to the Plane, on which the Shadows are receiv'd) nor are *Dials* exact, plac'd any where but at the Centre; where, and where only, the *Stile* truly anſwers to the Axis of the Globe, and the *Planes* themſelves, to the Planes of the *Great Circles*, which they repreſent.

But to proceed. It is upon the Account of this ſuppos'd Infinite Diſtance of the Eye; that all ORTHOGRAPHICK Projections are deſign'd by *Parallel* Rayes. Indeed in Nature, there is not, nor can be any ſuch thing, as *Parallel Radiation*; either from a REAL, or FICTITIOUS Radiant, ſuch as is an *Eye*; but the *Angles* becoming Indefinitely *Small*, and therefore Inſenſible, when the *Diſtance* is Indefinitely *Great*; we therefore take the Projecting Rayes in this Caſe, as Parallel, and proceed accordingly.

From hence it is, that in Projections of the *Sphere* this way, all Circles both *Great* and *Small*, the Planes of which, are not at Right Angles, to the

Plane of that Circle, on which the Projection is made; do all fall into the Form of *Ellipses*.

All *little* Circles, as also all *great* ones, which are perpendicular to the said Plane; (such as are the *Equator*, *Ecliptick* and *Horizon* in the Common *Analemma*) are represented by *strait Lines*. Nor can there be any sort of Line, *Circular* here; except only the Periphery of that Circle, in a Line drawn thro' the Pole of which, the Eye is suppos'd to be plac'd, at an Infinite Distance; Or in other Words; the Circle we project upon: as (for Example) the *Solstitial Colure*, in the *Instrument* just now mention'd.

From hence likewise it is, that all *Arches*, being projected, into their *Right Sines*; the Line of *Sines* is of so necessary Use, in describing and solving Problems, by this Sort of Projection. There are many Useful, and Noble Projections of the Sphere, made this way; and particularly very curious, (I won't say the most practicable) Constructions of *Dials* to be drawn from thence. Yet it must be confess'd, that as the nice Description of *Ellipses*, is a troublesome and laborious Practice; so there is an Inconvenience in that Respect, attending an *Orthographical* Projection, where a Problem requires an *Ellipsis* to be describ'd; which some of the very *Fine* ones do; tho' most of the useful *Vulgar* ones do not, but may be done by Right Lines and Circles only.

Nor should we omit taking Notice, of that particular Inconvenience likewise, in Orthographical Projections, *viz. The extream narrowing and crowding together of the Parts, toward the outside*; which is the unavoydable Consequence of the *Parallelism* of the Rayes: As common *Geometry* will convince any one, that divides the Circumference of a Circle in-

to

to a good Number of equal Parts, and draws Chords, thro' the opposite Correspondent Points.

This Sort of Projection, by *Parallel* Rayes, is very useful in other Cases, besides that of designing the Circles of the *Sphere* upon a Plane, for *Astronomical* Purposes.

In *Military Architecture*; the Draughts of *Fortifications*, are made this way, not only with more Ease and Expedition, but with most Convenience and Advantage too.

In *Civil* Architecture, *Orthography* properly signifies the upright Delineation of the *Front*: Thus *Vitruvius* defines it, *Orthographia est Erecta Frontis Imago, Lib.* 1. And by *Front* I presume is commonly intended, all that can be seen *directly*, at one single View; whether *inward* or *outward*, whether consisting of *one* Plane only, or of *more*. But this is a more *restrain'd* Sense and Application of the Word; for it denotes in general, a Delineation or Designation by *perpendicular* Lines; which comes up to the true Purport *of the Term* ὀρθογραφία. And it is after this way, that the *Plans* and *Elevations* of Buildings, are ordinarily drawn.

The *Ichnography* (or Plan) *ex. gr.* is an Orthographick Projection, on the *Ground Plane*; or, which is the same thing; 'tis the Section by a Plane parallel to the Horizon.

The *Profile*, is the same Sort of Projection upon a *Vertical* Plane, parallel to that, by which the Body is suppos'd to be *cut* through. Sometimes the *entire Section* it self (in which not only the bare out-Lines, but also the Thickness of the *Walls* appears) is represented this Way.

All these Projections are design'd, by Perpendiculars let fall, from the several Points of the Object,

to

to the Plane or Table, on which the Figure is to be drawn. For which Reaſon they muſt all of them, neceſſarily be *Similar* to their reſpective *Primitive Figures* ; being made (as they are ſuppos'd to be) on Planes *parallel* to thoſe, in which the *Originals*, or *Primitive Figures* are conceiv'd to lie. Farther, tho' *Height* and *Thickneſs*, may well be repreſented this Way ; yet there can be no Expreſſion of *Depth* or *Profundity*. The Nature of the Projection, will not allow any Repreſentation of *this Dimenſion*. However (as *Vitruvius* intimates, *Lib.* 1. *Ch*, 2.) it may be *allowable* to remedy this, by *Shading or Colouring* what is thus deſcrib'd *Orthographically* upon a Plane ; by which Means the Elevation and Depreſſion, and ſo the due Diſtinction of Parts, may be exhibited ; tho' it can never poſſibly be done, by the *bare Lineaments*, or Geometrick Deſign,

But to go on with our Diſcourſe.

The STEREOGRAPHICK Projection, comes next to be conſider'd. This is that, which is ſaid to be, *Ex Oculi contactu*, becauſe the Eye in this Sort of Projection, is conceiv'd to be poſited, on the very Surface of the Body or Figure to be projected. And there is this particular Advantage ariſing from thence, *viz. That, in the SPHERE, (about which this Projection is principally converſant) all the Parts are ſeparately and diſtinctly repreſented ; and that there is no one Point* (excepting that only where the Eye is plac'd) *whoſe Projection coincides, with the Projection of another Point.* For the Rayes drawn from the Eye, to the Points of the Spherick Surface, will cut the Plane on which the Projection is made, each in its own proper diſtinct Point. Indeed, in the Caſe of Bodies, that are contained under *Rectilineal-figur'd* Surfaces ; *there the Projections of ſeveral Points*

will

will be coincident with one another, and that becauſe of the Rectilineal Surfaces; as if *ex. gr.* the Eye were plac'd in one of the *ſolid* Angles, of either of the *Regular* Bodies; the Projections, of the ſeveral Points of thoſe Surfaces, whoſe Angles compoſe the Solid Angle where the Eye is fix'd, will be coincident with one another, becauſe the ſaid Points lie all in *ſtrait* Lines. But in the Sphere, or other Solid, contain'd under a *Curve* Surface; it will be otherwiſe. It is from hence, that this Projection has its Name of STEREOGRAPHICK; becauſe not only the *Ambit* or *Outſide* of a Body is this way deſcrib'd, but the τὸ ϛερέωμα, the *Solidity*, or entire Content of it: As the Geometry of Solids, is for the ſame Reaſon called ϛερεομετρία.

To give the compleat and *entire* Figure, of a Body thus on a plane Superficies, is the peculiar Property of the *Stereographick Projection*; for neither the *Orthographick* nor the *Scenographick*, can poſſibly do this.

Beſides; the Parts of the Projecture, in going from the Centre to the Circumference, which in the *Orthographick* Projection, are ſo crouded together; that they are the leaſt fit for Uſe, where many times they ought to be of the moſt Uſe; theſe here, *are gradually augmented*, and that with no very exceſſive Increaſe, till we come to a Hemiſphere; after which they are indeed, more immoderately augmented.

But then (which is likewiſe not only a noble Property, but a moſt conſiderable Eaſe and Advantage in this Projection, is, that) all the Circles of the Sphere, both *Small* and *Great* (except thoſe Great ones only which paſs thro' the Eye, and which are deſign'd by Right Lines) are repreſented here by *Circles*; and that as none of them elſe can be *ſtrait*

Lines,

Lines, so neither can they be *Ellipses*; as they will be (some of them, both *Great* and *Little* ones) in the *Orthographick* Projection. Farther, all Arches being projected here, into their *Semitangents*; that *Line* becomes of as standing Use in this, as that of *Right Sines* is in the *Orthographick* Projection.

The *Inverse* of the STEREOGRAPHICK Projection; is that which is commonly call'd the GNOMONICAL (as being that on which the ordinary Description of *Dials* is founded.)

I call it the *Inverse*, of the STEREOGRAPHICK, because of the *Reciprocal* Positions of the *Eye* and the *Plane*, in these two Projections.

For as *there*, an Eye plac'd somewhere on the Circumference of the Sphere, projects upon a Plane passing through the Centre; so *here*, an Eye plac'd in the Centre, projects upon a Plane, *touching* the Surface of the Sphere. Upon this Score 'tis, that Arches are here projected not into their *Semitangents*, as in the other, but into their *Tangents*,

All *Great* Circles fall into *strait* Lines.

All *Little* ones, parallel to the Plane of the Projection, come into *Circles*; and the rest, according to their various Positions, into the other *Conical Sections*.

This Projection, being not so vulgarly talk'd of, as the rest; I thought it would not be amiss, to give a little Explication of it here, in a *Figure* drawn for that Purpose. (See *Fig.* 37.)

Conceive the Sphere, whose Centre is O, and which is touched by an *Infinite Plane* in A, to be cut thro' its *Centre*, and the said Point of *Contact*, by another Infinite Plane; by which Means, the Great Circle which appears here, will be produc'd by the Section of the Sphere; the Infinite Line DAG for the

the Common Section of the two Infinite Planes, and the other Right Lines drawn in the Figure, will be the common Sections of the Planes of the Circles of the Sphere, both Great and Small, by the aforesaid cutting Plane.

It's plain that the *Great* Circles SX, QW, VR, &c. are projected into *Right Lines*; as all passing thro' the Eye at O. If the *Little* one KI be parallel to DG, then BE, is the projected Diameter of a *Circle*. But CE, into which PI is projected, is of Necessity, the longest Axis of an *Ellipsis*; and so of all other *little* Circles, drawn under the same Conditions.

For the Triangles COE, POI, can never possibly be *Similar*; the *latter* being ever an *Isosceles*. So that there can be no *subcontrary Section* here; and therefore no *little* Circle, can fall into a *Circle*, if it does not lie parallel to the Plane DAG. Such a prodigious Difference, does the bare shifting the Place of the Eye, in these two Projections make; that whereas in the STEREOGRAPHICK, we have nothing but *subcontrary Sections*, in the GNOMONICK, we have none at all.

The Circle LN will be an *Hyperbola* upon the Plane DG, which cuts the Side ON of the *Cone* LON, produced beyond the *Vertex* O. The Circle HM will be a *Parabola*; for I suppose TOM to be parallel to DAG. And so of the rest: A Man may at Liberty determine the *Positions* of his little Circles, and so see what Sections they will be, when *Gnomonically* projected. I could shew a Method, something peculiar, for describing these Curves; but that's not my Work here, and besides those Practices are common enough; nay, 'tis as common now a-days for People to do them, as 'tis for them, not to understand one Word of the Demonstrations.

monstrations. This same Figure, will serve to shew the Grounds of another pretty curious Speculation in these Matters; and that is, *What Conick Sections are described by the Shaddows of the Stiles of Dials, at any time of the Year, in any given Place.*

For suppose VR the *Axis* of the World, QW the *Equator*, LN some parallel of *Declination*; DAG, the *Horizon* of any Place.

The Angle DFV is the *Latitude*, suppose $= n$ Degrees; the *present Declination* WN $= p$ Degrees; therefore the Angle NOF or NOR $= 90 - p$ Degrees. Now if $n =$ or $<$ or $> 90 - p$; the Shaddow of the Gnomon upon the Plane DF, will at *that Time*, describe a *Parabola*, *Hyperbola*, or *Ellipsis*; as is most obvious from the various cutting of the *Cone* LON, by the Dial-plane DAG. If n were $= 90$, the Section becomes a *Circle*; and the Place whose Horizon DG is, is the *Pole* it self. This may be express'd in particular Examples, for particular *Latitudes*, and any Dial-planes at Liberty.

In the last Place of all, the SCENOGRAPHICK Projection comes to be consider'd. This proceeds (as they say) *Ex Justo & Moderato Oculi Intervallo*; the one of the other two Sorts of Projection being *ex Contactu*, and the other *ex Infinita Oculi distantia.*

What this *Just and Moderate* Distance is, is not so easily determin'd, though many have given their Rules for the fixing of it. Indeed speaking *Universally*, it is not determinable, in the very Nature of things: That being a moderate Distance, with Respect to one *Eye* and one *Object*, which is not so, with Respect to another; so that there can

can be no settling that Point, but with Regard to those Conditions.

This Projection is of no Use, with Respect to the Representation of the Circles of the Sphere: ('Tis true, a Circle may be a Circle here; but it must be by subcontrary Position; unless it stands parallel to the Table.) But 'tis of most admirable Use in designing all Sorts of Solids and Surfaces; *Buildings*, *Walks*, *Rivers*, *Animals*, and in a Word, whatever appears in Nature, within the Limits of a proper Distance. And this it does the most to the *Life*, of any Sort of Projection whatsoever. 'Tis this Science, which teaches those pretty *Frauds in Vision*, which give us so much Pleasure, and make us even fond of being imposed upon. 'Tis from hence that *Painting*, *Sculpture*, and all the fine Arts of *Imitation*, derive their Force and Beauty.

And 'tis the Explication of this, in its *demonstrative* Grounds and Principles, as well as in all the necessary Branches of *Practice*, which is the Design of the following Treatise.

I know there are many large, and pompous Books, written on this Subject: In a great Part of which, the Authors have been free enough of their *Examples*, but too sparing of their *Demonstrations*; and some few others, have demonstrated much more, than they have shewn the Use of; nor are there those wanting, who have justly mix'd both these together.

In this little Book, I would hope that the *Mathematical* Reader, may find both as much *Demonstration*, and as much *Practice*, as may enable him to perform any Problem whatsoever, relating to these Matters, in which the Stress of the Solution is to

lie

lie upon *Geometry*, and not upon bare *Delineation*. The Art of *Colouring*, is quite another thing, and so is that of *neat and curious Drawing*; both which may be in great Perfection without the rigorous *Mathematical Part*, as the Mathematical Part may be without them. 'Tis this *Latter* that is my Business here in this Treatise, which if it serves in any Measure either to entertain those that are knowing this Way, or to inform those that are not; I have obtained my End.

ERRATA.

PAge 57. Line 26. Read *Prop.* IX. In the Corollaries of Prop. XI and XII, the Figures referr'd to, will direct the Reader when to read N, and when O: The Point O is intended to be in the Middle, p. 65. l. 13. Fig. 12. From p. 65. to *Prob.* IV. p. 82. the No. of the Scheme, is unity less than it should be. From p. 82, to 90, the No. is right; and from p. 90 to the End of the Book, is defective as before. Pag. 148. l. 22. dele, *And the Height of the Eye.* Pag. 149. l. 7. dele *by*. Pag. *ibid.* read l. 19, 20, 21, 22, thus; *The Height of the Eye which is supposed to be unknown, we will denote by the Line* H. *But what is chiefly wanted, is that particular Distance of the Eye*, &c. Pag. *ibid.* at the End of l. 27. after the Words, *of N to M* add, viz. *For the Eye's Height will easily be found, when this Distance is once determin'd*: just as 'tis at Corol. I. Prop. V. Pag. 162. Line 2. for *by that*, read *that by*.

DEFI-

DEFINITIONS.

DEFINITION. I.

PERSPECTIVE is an ART which teacheth how to delineate the true Appearances of Objects, upon any Superficies, for any given *Distance* and *Height* of the Eye.

DEF. II.

The *Perspective Table*, or Plane, is that, whereon the Picture of the Object is form'd, according to perspective *Rules*.

DEF. III.

The Geometrical, or ground Plane, is that whereon the *Perspective Table* is supposed to stand.

DEF. IV.

The *Height of the Eye*, is a Perpendicular let fall from it, to the Ground Plane.

DEFINITION V.

The *Distance* of the Eye is a Perpendicular let fall from it, to the Perspective Table.

DEF. VI.

The common Section of the Perspective Table, with the *Ground Plane*, I shall call the Ground *Line* (or *Section.*)

DEF. VII.

The *Horizontal* Line, is a Line in the Table, Parallel to the *Section* or *Ground Line*, and of the Height of the Eye above it.

DEF. VIII.

The *Principal Ray*, is the Line let fall from the Eye Perpendicular to the Table, and therefore is equal to the *Distance of the Eye from the Table.*

DEF. IX.

The *Distance* of any Point in the ground Plane, *from the Table*, is *a Perpendicular let* fall from the Point, to the *ground Line.*

DEF. X.

DEFINITION X.

Direct Parallel Lines, are ſuch as cut the ground Line or Section at Right Angles.

DEF. XI.

Oblique Parallels, are ſuch as are drawn cutting the ground Line or Section, at any oblique Angle whatſoever.

DEF. XII.

Tranſverſe Lines, are thoſe which cut the *Direct* Lines at Right Angles.

DEF. XIII.

Radial Lines, I call ſuch as run up from any Points in the *ground* Line, to any Perſpective *Focus*, whether the Point of Sight, or *accidental* Point, *&c.*

DEF. XIV.

By *the Point of Sight*, is underſtood that Point in the Table, in which all the *direct* Parallels ſeem to concur. How it is determin'd, we ſhall ſee afterwards.

DEFINITON. XV.

The *Accidental Point*, is a Point which bears the ſame Relation to ſuch Parallels as are *oblique* to the Ground Line, as the *Point of Sight* does to thoſe which are *perpendicular* to it: That is, as the *Point of Sight* is that in which all the *direct* Parallels ſeem to concur; ſo in like manner, the *accidental Point*, is that in which any *oblique* Parallels do appear to the Eye to meet and unite. So that tho' (ſtrictly ſpeaking) there be but one *Point of Sight*; yet, there are innumerable *accidental Points*, even as many, as there are different Degrees of Obliquity, in which the *ground Line* or *Section*, may be cut by the forementionꝰd *oblique* Parallels.

D E F. XVI.

The *Point of Diſtance*, is a Point in the Horizontal Line of the Table, determin'd therein, by laying off from the *Point of Sight*, either way, the Eyes Diſtance from the Table.

D E F. XVII.

A Point of *Incidence*, is a Point in the ground Line, determin'd by a *Perpendicular*, let fall from any Point in the *ground Plane*, thereto.

DEFINITION XVIII.

The Perſpective of any Point, is there, where a *viſual* Line drawn from the Eye interſects the Table; or 'tis the Interſection of the Plane of the Table, by a *viſual* Line drawn to that Point.

DEF. XIX.

The Perſpective of a *Line* either *Strait* or *Curve*, is the common Section of the Plane of the Table, and the *viſual* Superficies (whether Plain or Curve) whoſe Baſis is the aforeſaid Line.

DEF. XX.

The Perſpective of any *Plane Figure*, *Rectineal*, or *Curvilineal*, is the Section of the Cone or Pyramid (whoſe Vertex is the Eye, and Baſis, the Figure propos'd) by the Plane of the Table.

DEF. XXI.

The Perſpective of a *ſolid* Figure, is the Aggregate of the Perſpectives of all the Planes (whereof that Solid is compoſed) aptly and truly ſet together, upon the Plane of the Table.

DEFINITION XXII.

The *Optick* Angle, under which any Object appears, is that which is comprehended under 2 Lines drawn from the Center of the Eye, to the two Extremities thereof.

According as *this Angle* is bigger or leſs, ſo we commonly ſuppoſe things to appear bigger or leſs to us. And it is moſt certainly true, that they do ſo, in Varieties of Caſes: But that they do ſo in all Caſes, is as certainly falſe. As generally as the Rule paſſes amongſt the *Opticians*, it is not univerſally true, that an Object which is ſeen under a *bigger Angle* than another Object is, does therefore *appear bigger* to the Eye. And this will be ſufficiently made out, by the following Demonſtration, which is *Experimental* and *Ocular*.

Suppoſe that there were placed at A, (Fig. 2.) the Eye of a Spectatour, in ſome *long Room or Walk*; the Eyes Height being A B, and B K the *Walk* or Ground Plane, parallel to the Horizon. Let the Heighth of the Spectatours Eye, *viz.* A B, be laid off in the Ground, from his *Foot* at B, to N; ſo that B N = B A. Then ſince A B N = 90°, 'tis plain that BAN = BNA = 45°. Thereſore B A N > N A I, N A V,

N A K,

NAK, or any other Angle, comprehended between the Horizon BK, and a Ray drawn from the Eye at A. But it is plain in *Fact and Experience*, that the Diſtance BN ſhall not appear *equal*, or *bigger*, but *leſs* than NI, NV, NK, *&c.* in the Horizontal Line: And yet BN is ſeen under a bigger Angle than any of all the Diſtſtances, NI, *&c.*

Therefore it is not univerſally and abſolutely true, that every Object which is ſeen under a bigger Angle than another, does therefore *appear bigger* than that other Object does. *Q. E. D.*

COROLLARY

Therefore neither is it univerſally true, *That Objects muſt be ſeen under equal Angles, in order to their appearing equally Big.*

For here, the Diſtance NI *ex. gr.* appears as big to the Eye, as BN does, and and yet the Angle NAI, is much *leſs* than BAN. Nay, (according as the Diſtance is taken) NI ſhall appear prodigiouſly bigger than BN; tho' the Angle (as is obſerved) be ſtill demonſtrably *leſs*: So that there is no Doubt of the Truth of the Corollary.

And therefore I muſt ſay farther, That ſince this Rule (*of Objects appearing equally*

Bigg, which are ſeen under equal Angles) is ſo frequently made uſe of in moſt Books of *direct Opticks*, and ſo many things are grounded upon it, as we find there are; an accurate Enquiry ought to be made, in what Caſes it holds true, and what not. In the mean time, I ſhall offer a few things concerning it, which Reaſon and Obſervation together, render me pretty well ſatisfied of the Truth of.

1. That the Rule holds true, when it ſpeaks of Spaces or Intervals, taken in any Line, on each ſide *a Perpendicular, let fall from the Eye to the ſaid Line, and equally removed from that Perpendicular.*

2. That it is true likewiſe, when it ſpeaks of Lines *Parallel to each other*, and which lie in ſuch a Poſition to the Spectatour, that a Line drawn *directly forwards* from his Foot, croſſes thoſe Parallels at Right Angles. Let the Parts veiw'd, lie in *equal Circumſtances of Diſtance*, from this Croſs Line; and then, ſuch Segments of theſe Parallels as are intercepted between *viſual Rayes making equal Angles*, will without doubt appear of *equal Bigneſs*, when ſurvey'd with a free Caſt of the naked Eye.

3. That the Rule is always falſe, when it is apply'd to Spaces, taken in one and the ſame Right Line, one and the ſame way; *by which I mean, only their being taken on*

one

one and the ſame Side of a Perpendicular, let fall from the Eye. Thus for Example, it was in the Caſe of the Demonſtration produced: And it would be the ſame, if we were to look in *Breadth* or *Height*, as well as in *Length.*

I do not exclude other Caſes, beſides theſe which I have mention'd, from being Inſtances either of the Truth or Falſehood of the Rule. Theſe are only ſuch, as are the moſt common, and the moſt eaſie to be try'd.

SCHOL. I. *To the Preceding Definitions.*

Tho' the Perſpective Table may be plac'd in *various Poſitions*, with Reſpect to the *Eye*, or *Ground Plane*, whereon it ſtands, yet it is commonly imagined to be *perpendicular* to the Ground Plane; this Poſition being of all others, the moſt *ready and familiar to us.* Tho' we ſhall ſhew in the enſuing Part of this Treatiſe, how the Rules of drawing Pieces of Perſpective, upon Tables *Perpendicular* to the Horizon, may be accommodated to Tables, *in any other given Poſition* whatſoever.

In like manner, tho' we may conceive the Appearances of Objects, to be delineated upon Curve Superficies, whether *Convex* or *Concave*, as well as on *flat* or *plain* Superficies;

Superficies; yet for the ſame Reaſon as before, the Perſpective Table is moſt commonly taken to be a *Plane.*

SCHOL. II.

Hitherto we have only mention'd ſuch Lines (amongſt the Definitions) as lie in *the Horizontal*, or Ground Plane *beneath* the Eye. But as we may conceive an infinite Number of other Planes, *ex. gr.* Horizontal ones *above* the Eye, *Direct*, *Declining*, *Inclining*; and ſuch as do both *Decline*, and *Recline*, or *Incline* together; ſo the various Sorts of Lines which may be drawn in theſe Planes, are alſo to be conſidered, and will all fall under the *General Rules*, hereafter to be deliver'd.

EXPLICATION to the foregoing Definitions.

Fig. 1. W K L the Geometrical or Ground Plane.
A B D C the Perſpective Plane or Table.
H the Place of the Eye.
P H the *Heighth* of the Eye, = E F in the *Table.*
H E the principal Ray, or *Diſtance* of the Eye from the Table, which is = P F in the *Ground Plane.*

DY,

DY, Cn, OT, yS, *direct* Lines.
VW, XZ, Two *transverse* Lines.
E the Point of Sight.
mR, rs, tU, MQ, oblique Parallels.
Dt Mn OF ymrdC, the *Section* or ground Line.
Pn, a Line from P parallel to the oblique Lines tU, MQ.
Pd Parallel to the oblique Lines mR, rs,
fAGEg BL, the *Horizontal* Line.
HG, a Line from the Eye parallel to Pn.
Hg, a Line from the Eye parallel to Pd.
G, g, the *accidental* Points, relating to the oblique Parallels
tU, MQ, and mR, rs, respectively.

PROP. I.

The farther Parallel Lines are produced from the Sight, the nearer they seem to approach to each other; provided the Eye be placed any where, *between* the said Parallels.

This is true, whether the Eye, be *in the same Plane*, with the Parallels proposed, or whether it be *raised above*, or *depressed below them*.

1. Let the Eye at A, be placed in the *same Plane*, with the Parallels BK, RM. (Fig. 2.)

CON-

CONSTRUCTION.

Draw DC, LI, MK, &c. Perpendicular, as alſo AQ Parallel to the Lines BK, RM.

DEMONSTRATION.

The $\triangle$ls LAI, MAK (whoſe common Vertex is A) have the Baſe LI = the Baſe MK; but Perpendicular AQ > Perpendicular AP (by Hypoth.) therefore Angle MAK < LAI, therefore MK appears < than LI, and the Parallels appear *nearer* to each other in the Points M, K, than in L, I. *Q. E. D.*

2. Let the Eye at B, be placed *above* or *below* the Plane, in which the Parallels AH, DK, are drawn. (Fig. 3.)

CONSTRUCTION.

Croſs the Parallels, with the Perpendicular Lines AD, EG, HK, &c. From B, let fall BC perpendicular to the ground Plane, and carry out the *viſual* Rayes BE, BG, BH, BK. From C, draw CI Parallel to AK, DK; and tho' the Lines BC and CI, conceive a Plane to paſs, whoſe common

common Section, with the *Ground Plane*, will be that same Line C I; and with the two *visual* Planes, will be B F and B I. *Lastly*, join the Points C and K, with a Right Line.

DEMONSTRATION.

Because (*Constr.*) B C is Perpendicular to the Plane A D H K, therefore the $\triangle^{ls}$ B C I, B C K are Rectangular at C. Farther, since (*Constr.*) IK is perpendicular to C I, therefore the $\triangle^{l}$ C I K is also rectangular at I.

Therefore, $BK^q = CK^q + BC^q = CI^q + IK^q + BC^q = IK^q + BI^q$. So that $BK^q = IK^q + BI^q$. Therefore the Angle BIK is a Right one, and therefore BIH is a Right one. After the same manner, it may be demonstrated, that the Angles BFG, B F E are Right ones. Therefore, in the Rectangular Triangles, BIH, BFE, because the Base $HI = EF$ (*Hypoth.*) and $BI > BF$ (for by Hypoth. $CI > CF$) therefore shall Angle $EBF > HBI$.

For the same Reason, $GBF > KBI$. Therefore, $GBE > KBH$. Therefore the Parallels seem nearer to each other, in the Points H, K, than in E, G. *Q. E. D.*

COROL.

COROLLARY I.

It is certain therefore, that Lines which are *really* Parallel, cannot be seen *Parallel.*

For to be seen *Parallel*, they must appear *Equidistant* in all their Parts; whereas we are assur'd by the foregoing Demonstration, that they seem continually to approach each other: That is, they appear *Converging*.

COROL. II.

Paralllel Lines indefinitely produc'd, will appear to the Eye, to meet in a Point; because the *Optick* Angle, subtended by the *Interval* or *Distance* between them, at that Indefinite Prolongation, will become *Insensible*, or of no Quantity in a *Physical* Sense.

PROP. II.

The Rate at which Parallels seem to converge, is determin'd *by the Reciprocal Proportion of the Tangents of the Optick Angles, to the Perpendicular Distances of the Eye from the said Parallels.*

1. If the Eye be in the *same* Plane: (Fig. 2.) Let the Parallels be R M, A Q, and

and the Eye at A, and the *Rayes* A M, A L, A Q, cut the Line D C, in F, E, O, respectively.

From Similar $\triangle^{ls}$ AMQ, AFO,
AO : AQ :: FO : MQ,
From Similar $\triangle^{ls}$ ALP, AEO,
AO : AP :: EO : LP,
Therefore AQ : AP :: EO : FO;
But EO : FO :: T, EAO : T, FAO.
Therefore AQ : AP :: T EAO : T, FAO.
Q. E. D.

2. If the Eye be out of the Plane, (Fig. 3.) let the Eye be at B, the Parallels AH, CI, and the *visual* Rayes, as before.

In the Rectangular $\triangle^{l}$ BIH,
BI : HI :: R^{d}. : T, HBI,
In the Rectangular $\triangle^{l}$ BFE,
BF : EF :: R^{d}. : T, EBF,
Therefore BI : BF :: T, EBF : T, HBI.
Q. E. D.

COROL. I.

Hence we see how the visible Magnitude of an Object increases or decreases, in its various Approaches to, or Removes from the Eye, *viz.* thus, That *the apparent Diameters; are reciprocally as the Distances from the Eye.*

COROL. II.

COROL II.

The Eye, in the ſame Poſition, looking at the ſame Object; removed to various Diſtances, EF, HI, there is *a leſs Proportion*, between the Tangents of the Optick Angles, when the Eye is placed *above* at B, than when it is *below* at C.

CONSTRUCTION.

Draw FL Parallel BI.

DEMONSRATION.

T, ECF : T, HCI :: IC : CF, by N°. 1.
IC : CF :: IB : LF, by Similar $\triangle^{ls}$ BIC, LFC.
Therefore, T, ECF : T, HCI :: IB : LF.
But T, EBF : T, HBI :: IB : BF, by N°. 2.
And IB : LF > IB : BF. Therefore,
T, ECF : T, HCI > T, EBF : T, HBI.
Q. E. D.

COROLLARY II.

Parallel Lines ſeem to *converge faſter*, to an Eye poſited in the *ſame Plane* with them, than to an Eye raiſed *above*, or depreſſed *below* that Plane.

SCHOL.

SCHOL.

In arguing here upon the Appearances of *Parallels*, we have taken the Truth of the old Maxim for granted, *That a Space seen under a less Angle, appears less, and under a bigger, greater.* And I believe the Case is so plain, that there will be little Dispute about the Truth of it here.

PROPOSITION. III.

If the Eye be seated any where without the Parallels, they will seem to go farther from each other (or their Intervals to widen) *to a certain Term of Distance; and after that, continually to approach each other.*

CONSTRUCTION.

Let the Parallels LA, KC, (Fig. 4.) whose *Distance* EG, is bisected in F, and and FQ *drawn* parallel to them. Let the *Eye* be at D, in the Line EG produced. Upon the Center F, with the Radius FD, strike the Circle DIH. On each other Centers, as M and B, taken at Liberty in the Line RF, and with the Radii MD, BD, strike other Circles; the *former* of which, imagine to cut the Parallels in the Points L, K; and the *latter*, in N, P. It is plain, that the Lines, LK, NP, shall each be

equal to E G. For ſince the Centers M, B, are taken in the Right Line F R, which Line *perpendicularly* biſects E G, in the Point F; it is evident that the Lines, *joyning the Interſections* of theſe Circles, with the Parallels, *viz.* I H, L K, N P, ſhall be ſo many *equal Chords*, in theſe ſeveral Circles.

Draw the Lines D I, D H, D N, D P, D L, D K.

DEMONSTRATION.

Becauſe the Angle D F B is ∟r, therefore in the $\triangle^l$ D F B, the Side D B > D F. So in the $\triangle^l$ D F M, for the ſame Reaſon, D M > D F. Therefore the Circles, whoſe Centers are B and M, and Radii DB, DM, are greater than the Circle, whoſe Center is F, and Radius D F. Since therefore the Chord I H = N P = L K, and the Circle D I H is the leaſt of all the Circles; alſo the Angle I D H ſhall > N D P, or L D K. And therefore the Parallels appear fartheſt aſunder in IH, and from that *Limit* ſeem to approach to each other, both ways, *viz.* on one ſide towards N P, and on the other towards L K. *Q. E. D.*

SCHOL.

As it has been ſhewn in ſome Inſtances, that Lines which are *truly and ſtrictly* parallel, will ſeem not to be Parallel; ſo it

may

may alſo be ſhewn *how*, and in *what Circumſtances*, Lines which are *really not Parallel*, may yet appear either *Parallel*, or elſe *as Parallels*. For we muſt take Care here, not to confound together two Notions, which in the Nature of Things are very different, *viz.* being ſeen *as Parallels*, and being ſeen *Parallel*. For two Lines to be ſeen *Parallel*, is for thoſe Lines to appear equally diſtant in all their Parts (as was hinted before, at Cor. 1. Prop. 1.) But for two Lines to be ſeen *as Parallels*, is for thoſe Lines to appear, *after the manner of Parallels*, or to appear *as Parallels uſe to appear*; that is, to ſeem inclining and converging towards each other, after the manner that ſuch Lines ſeem to do. Thus 'tis demonſtrable, that *two Right Lines, which are not parallel* to each other, may yet appear to the Eye, (diſpoſed at a *certain Diſtance and Poſition*) as Parallel Lines uſe to appear.

For if thoſe Lines be produced, till they concur, and the Angle contained between them, be biſected, and the biſecting Line be croſſed at Right Angles, by two Right Lines, terminating on each Hand, in the converging Lines firſt given; *then a Circle being deſcrib'd, about the Trapezium thus form'd, and two Lines to touch this Circle, being drawn from the Point, where the Lines at firſt produced, met one another; and laſtly, the two Points of*

Contact being joined by a strait Line: The Distance between the *Point of Concourse* aforemention'd, and the Point where the Line *joyning the Contacts*, crosses the *bisecting* Line, is the Diameter of a Semi-Circle, which will be the *Locus* requir'd; or such, that the Eye being placed in any Point thereof, the given converging Lines shall appear to it, *as Parallels* would appear. The *Analysis* of this Problem, evinces, That the *Locus* is a *Circle*, as also how it is to be *constructed*; but as the thing it self is not essential to my Purpose, so neither is this a proper Place for such Enquiries.

Again, it might be shewn, in like manner, how two Lines not parallel, one being a *Right Line*, and the other a Curve; may, notwithstanding, *appear Parallel*, or equidistant in all their Parts. *For if a strait Line be drawn in a Plane, and some fixed Point taken therein, as a Pole or Center, about which, the said Line revolves, keeping still in the said Plane, while, at the same-time, another Right Line making any oblique Angle, with the Plane, revolves about the same Point, describing thereby a Conical Surface*: also if a *Second* Plane be conceived to be drawn, either *Perpendicular* or *Oblique* to the former Plane, by which Means, some one or other of the *Conick Sections* is produced, then 'tis demonstrable, that to the Eye, posited in

in the *Pole* (which is also the *Vertex of the Cone*) all those *unequal Intervals*, contain'd between the Conick Curve, and *that Right Line*, which is the common Section of the two aforesaid Planes, *will appear of equal Bigness*, provided the common Sections of the Planes of *Visual Rayes*, with the *Second* Plane abovementiond, be all *Parallel* one to another. *N. B.* When the Second Plane is *perpendicular* to the First, the Curve form'd, will be an *Hyperbola*; when *Oblique*, a *Parabola*, or *Ellipsis*.

PROP. IV.

All Planes seated above the Eye, seem to sink the more downwards, the further they are produced: Those that are below the Eye, seem to rise upwards; those on the Right Hand to approach to the Left, and those on the Left, to the Right.

CONSTRUCTION.

Let the Eye be A, (*Fig.* 2.) its Heighth AB, a Plane *above* the Eye RM, a Plane *below* the Eye BK, the *Table* DC. Draw the Rayes AL, AM, AI, AK: Then shall the Points L, M, appear in E, F, and the Points I, K, in G, H.

DEMONSTRATION.

In the Rectangle Triangles RAL, RAM, whose Base BA is Common, the Perpendicular

dicular, RM > RL, therefore the Angle RAM > RAL, therefore AM falls *without* the Line AL, and therefore cuts the Table DC in a Point F *lower* than E.

In like Manner, it will be proved that the Point K appears *Higher* in the Table than I.

And so it may be prov'd by the same way of arguing, That Planes lying on the *right* Hand of the Eye, seem to approach nearer and nearer to the *Left*; as those also on the Left, to approach to the Right.

For we need only to suppose the Eye A, to be plac'd between two Planes, as BK on the *Right*, and RM on the *Left*. Therefore, *&c.* *Q. E. D.*

SCHOL.

The Truth of the Proposition may otherwise thus appear. Since any visible Point as *M*, appears not to the Eye in the same Place that it really is in, but in some other Place in the same Ray *AM*, nearer, as at *N*; so likewise, since the Point *T*, is not seen in *T*, but somewhere nearer, as at *n*: For this Reason, the Space *TM* shall appear in *nN*, that is, *falling downwards*.

And for the same Reason, the Space *VK* shall appear in *sS*, rising upwards towards the Eye.

But

But 'tis to be obſerv'd, that as the Points *L* and *I*, are not ſeen there, but ſomewhere nearer in the ſame Rayes *AL*, and *AI*; ſo conſequently the Space *LM*, cannot appear in *LN*, nor *IK* in *IS*, and therefore the Repreſentation of the Planes *LM*, *IK*, cannot be the Lines *Ln Nn*, *Is Ss*, as they are here drawn from the Points *L* and *I*: Becauſe, I ſay, the Points *L* and *I*, being not ſeen where they are, but nearer to the Eye *A*; the Lines *Ln Nn*, *Is Ss*, cannot begin at the Points *L* and *I*, but at ſome other Points between them and the Eye *A*. As for the *Species* of theſe Lines, it's manifeſt they cannot be *ſtrait* Lines, but *Curves*, approaching continually nearer and nearer to the Line *AQ* produc'd.

Which Line *AQ*, will be as a common *Aſymptote* to them.

The *Nature* of theſe Lines is to be determined by *Obſervation* and *Experiment*; namely, when it ſhall be determined at what Diſtances the Points T, M, do appear in the Rayes AM, AT, from their true and real Places; that is, how far the Points n, N, *&c.* are from the Points T, M, *&c.*

From the Propoſition before demonſtrated, we may ſee the Reaſon of ſeveral Appearances. which are very common.

COROL. I.

The *Floors and Pavements* of Buildings (eſpecially thoſe that are very long) ſeem to riſe upwards, towards the Eye of the Spectatour, that enters them.

COROL. II.

For which Reaſon, in *Churches*, *ex. gr.* the Pavement, in going from the *Door*, towards the *Altar*, need not be raiſed above the Level, ſo that a Perſon ſhould continually aſcend in approaching towards the *Latter*, from the *Former:* Becauſe, beſides that there is already an Aſcent, which proceeds from the Principles of *Opticks*; which therefore ought not to be made yet more conſiderable, by an actual Elevation of the *Floor*; there would be this farther Inconvenience, in raiſing it above the Level, *viz.* That the Orders of the *remoter Columns*, being therefore neceſſarily ſhorter than thoſe nearer the Eye, they would be ſo immoderately ſhortned *in the Appearance*, as to offend the Spectator's Eye very much at his Entrance.

COROL. III.

The *Roofs and Cielings* of Buildings, appear gradually to ſink down towards the Eye.

COROL. IV.

And therefore, any *Roof or Contignation* ought to be ſo much the Higher, by how much, the *Area* which lies under, extends it ſelf farther in *Length*.

For otherwiſe, at a conſiderable Diſtance, it would ſeem to hang down upon the very Ground it ſelf.

COROL. V.

Long Rowes of *Columns* or *Pilaſters*, *Trees*, *Walls*, and *the Sides of Buildings*, contract themſelves to the Eye, and ſeem to grow narrower and narrower.

COROL. VI.

And for this Reaſon, in order to make *Proſpects* of this Kind truly pleaſing and agreeable; Care ſhould be taken, that the *Breadth or Wideneſs* of them, be duly proportion'd to the *Length* they are deſign'd to be of.

SCHOL.

A Man may at any time, experiment the Truth of the foregoing Corollaries, in a long *Portico* or *Piazza*, adorn'd with Orders

ders of Pillars. There he may ſee, how the *Pavement* ſeems to riſe, the *Roof* to ſink down towards the Eye, and the *Side-walls* to incline to each other; and all verging to a Point: which Phænomenon was moſt accurately deſcrib'd by the *Philoſophical Poet*, in thoſe excellent Lines.

Porticus *æquali quamvis eſt denique ductu*
Stanſque in perpetuum paribus ſuffulta Columnis,
Longa tamen in parte abſumma cum tota videtur,
Paulatim trahit anguſti faſtigia Coni;
Tecta Solo *jungens, atque omnia* Dextera Lœvis,
Donicum (or *Donec*) *in obſcurum* Coni *conduxit* Acumen.

Lucret. Lib. 4.

COROL. VII.

The *Capitalls* of Pillars appear inclining downwards, and the *Pedeſtals* riſing upwards.

COROL. VIII.

The *Horizon* appears higher, than it really is. For, becauſe of the immoderate Diſtance between it and the Eye of the Spectator; it ſeems to be of an equal

Height

Height with the Eye it ſelf. And therefore every Spectator has a *different optical Horizon*, according to the different Altitude of his Eye above the Plane of the *real* ſenſible Horizon (which is a *Tangent* to the Surface of the Earth, in that Point where the Perſon ſtands.)

COROL. IX.

For the ſame Reaſon, the Convex Surface of the *Sea*, to an Eye placed thereon, appears differently *Protuberant* and *Curv'd* from what it is in it ſelf.

COROL. X.

It follows likewiſe, that if a Row of Columns *(ex. gr.)* all equal in Height, and Perpendicular to the Horizon, were diſpos'd in Order *beneath* the Eye; thoſe which are the *remoteſt*, would appear to be *lifted up higher*, in Proportion, than the reſt.

But if they were diſpoſed *above* the Eye, thoſe which are the *remoteſt*, would ſeem to be more *ſunk or depreſſed*, than the *nearer* ones.

For by the *Propoſition*, this is true of any Points (in theſe Magnitudes) which are terminated in the ſame Horizontal Line; therefore, it is true of all Points in them, terminated by Horizontal Lines; that is, of the whole Magnitudes themſelves.

SCHOL.

SCHOL.

From this *last Corollary*, ariſes another Conſideration which deſerves Regard, *viz. That Superficies, which are exactly plain and level to the Horizon, plac'd* ex. gr. *above the Eye*, muſt neceſſarily appear ſunk in and hollowed.

This infers the Reaſon and Uſe of thoſe *Scamilli*, whereof *Vitruvius* ſpeaks, as a Remedy to prevent ſome unpleaſing Appearances, in a piece of *Architecture*. Stylobatam *ita oportet exæquari uti habeat per medium adjectionem per* Scamillos impares; *ſi enim ad* Libellam *dirigatur*, Alveolatus *oculo videbitur*. *Vitruv. Lib.* 3. *Cap.* 3.

The ſame Conſideration is likewiſe of uſe in the ſhaping of *Images* and *Statues*, which are to be plac'd at conſiderable Heights above the Eye. For a Figure which ſhews all the exact Symmetry and Proportion, in the World, to the Eye, at one Elevation or Diſtance, will perhaps, loſe all thoſe Charms, and become downright ugly at another. So that in thoſe Caſes, *Art* is to conſult and ſee, what is to be *Added* or *Taken away*; that the great Ends of Beauty and Pleaſure may be provided

for,

for, according to the *Nature and Conditions of the Place* from which an Object is to be viewed.

Alia enim ad manum Species esse videtur, alia in excelso, non eadem in concluso, dissimilis in aperto; in quibus magni Judicii est Opera, quid tandem faciendum sit. Vitruv. *Lib. 6. Cap.* 2.

And again, *Cum ergo quæ sunt vera, falsa videantur, & nonnulla aliter quam sunt oculis probentur; non puto oportere esse dubium, quin ad Locorum Naturas aut Necessitates,* Detractiones aut Additiones *fieri debeant: sed ita ut nihil in his operibus desideretur.*

It was owing not only to Knowledge in in *Sculpture*, but to Skill in *Proportions*, and especially to the Knowledge of *Optical* Appearances, and the Reasons of them; that the celebrated *Phidias*, at once surpriz'd all the People of *Athens*, and triumphed over *Alcamenes*, who was his Rival, for Fame and Glory in the Art of Carving.

The *STORY* we have in *Tzetzes, Var. Histor. Chil.* 8. *Hist.* 193.

'THough these Persons were both
' of them excellent *Statuaries*, yet
' *Alcamenes* understood only the *Mechanick*
' servile Part of his Art; whereas *Phidias*
' being

'being well ſeen in *Geometry* and *Perſpe-* '*ctive*, knew how to render his Work 'compleat by the Rules of thoſe Sciences. 'Now, the *Athenians* having appointed a 'Statue of *Minerva* to be ſet up in the 'Market Place: Each of theſe Artiſts, 'was order'd to imploy his beſt Skill in 'the making of one. Accordingly, *Alca-* '*menes* made a Statue of ſuch charming 'Beauty, to an Eye which veiw'd it *at* '*a ſmall Diſtance*; that all the People at 'firſt Sight, adjudged him the Victory. 'And they thought themſelves ſtill more 'in the Right, when *Phidias*'s Work ap- 'pear'd. For, he conſidering at what '*Height* the Statue was to be plac'd, had 'ſhap'd it accordingly; making the Coun- 'tenance horridly diſtorted, and all the 'Limbs ſo diſproportion'd, that it look'd 'more like the Figure of a Devil than a 'Goddeſs. And the *Mob* (who never judge 'by Reaſon, and the Rules of Art, but by 'preſent Senſe) were well-diſpos'd to have 'made him ſenſible of their Reſentments, 'upon the Score of the Affront they 'thought was offer'd to *Pallas*, in making 'ſuch a filthy Thing to repreſent her: 'However, they houted him, and cried 'up *Alcamenes* for an Artiſt beyond Com- 'pariſon. And thus Matters ſtood (*Phidias* 'enduring the Perſecution of the ignorant

'Rabble)

'Rabble) till both the Statues came to be 'set up at the *appointed Height*. But then the 'Scene was quickly chang'd. All the soft 'Strokes and Graces of *Alcamenes*'s Image, 'quite disappear'd; as on the other Hand, 'did the rough and barbarous Features of 'that made by *Phidias*. So that now (both 'being view'd at the *proper Distance*) the '*Former* appear'd ugly, and the *Latter*, 'exquisitely fine and beautiful: And so '*Phidias*, (besides the *Prize*) went off with 'as much *Praise*, as before he had *Con*-'*tempt*, from the Common People.

Such Trials of Skill are sometimes seen in *other Arts* besides Sculpture; and there are more *Alcamenes*'s, and *Phidias*'s, besides those who contended at *Athens*.

PROP. V. THOR. V.

If the Object be a plane Figure, seated in a Position parallel to the Table; its Per*spective will be a plane Figure similar thereto.* (The *Picture* and the *Original*, will be like each other.)

Tho' this may easily be conceiv'd, for the Section of the *visual Pyramid or Cone*, by a Plane parallel to its Base; I shall, notwithstanding, demonstrate it *in Form*, for the

the sake of those who may desire to see a strict Proof, for all the Conlusions advanced to them, in this Science.

CONSRUCTION.

The Object (Fig. 5.) being the Figure DEFL; from the Eye at A, draw to the several Angles thereof, the visual Rays AD, AE, *&c.* by which Means the *optick Pyramid*, ADEFL is formed, and is also cut by the Plane of the Table NRKS Parallel to DEFL, the Section produced being BCGI. Draw LE, and IC.

DEMONSTRATION.

Because the two parallel Planes are cut by the Plane AEF; the common Sections FE, GC, shall be parallel. Therefore the $\triangle^{ls}$ AGC, AFE, are Similar. So likewise, are AGI and AFL, and for the same Reason ACI and AEL.

Wherefore, AG : AF :: GC : FE.
Also, AG : AF :: GI : FL.
Therefore, GC : FE :: GI : FL.
Also, AG : AF :: AC : AE.
And AC : AE :: CI : EL.
Therefore, AG : AF : : CI : EL.
Therefore, CI : EL : : GC : FE : : GI : FL. Wherefore, since the Sides of the

the $\triangle^{ls}$ CIG, ELF, are thus proportional, it follows, that they are *Equiangular.*

And thus may all the reſt of the $\triangle^{ls}$, whereof the *Original*, and the *Image* are compoſed, be ſhewn to be Similar.

Therefore the *polygonal* Figures themſelves are ſo, *Q. E. D.*

COROL. I.

The Object being any *Rectilineal* plane Figure, its Perſpective is *Diſſimilar* to it, when its Poſition is not Parallel to the Plane of the Table.

But if a *Circle*; its Perſpective may be *Similiar* to it, that it may be a Circle in ſome certain Poſition; tho' it does not lie Parallel to the Table, becauſe the *viſual Cone* may be cut *ſubcontrarily*, by the Plane of the Table.

Thus (Fig. 6.) ſuppoſe the *Object* DE a *Circle*, the *Table* GD, the Eyes *Diſtance* BD. We may determine from theſe *Data*, a proper *Height* of the Eye as FB, ſo that the viſual Cone DFE may be cut *ſubcontrarily* in CD, and conſequently the *Perſpective* be a Circle too. Take BA = BD, and then biſect the Line AE in H, ſo that

AH or EH $= \dfrac{DE + 2\,BD}{2}$; on the Cen-

D ter

ter H, and with the Radius AH ſtrike a Circle, cutting a Perpendicular from the Point B, in F; which will be the *Place* of the Eye *ſought*, and conſequently FB its *Height*. Joyn AF. The Angle AFE = ∟ (becauſe of the *Circle*) therefore ſince FB Perpendicular to AE, alſo the Angle AFB ſhall = BEF. But becauſe AB = BD (*Conſtruct.*) therefore BFD = BFA, therefore BFD = BEF. But BF parallel to GD, therefore BFD = CDF, therefore DEF = CDF, therefore the △^l FCD is Similar to the △^l FDE. Wherefore the Cone is cut *ſubcontrarily*, and the Perſpective CD is a Circle. *Q. E. D.*

COROLLARY II.

Hence may be found ſuch a Diſtance, of the *Object* or *Eye*, from the Table, that the Perſpective ſhall not only be *Similar*, but alſo in any given Proportion to the *Original*. *Ex. Gr.* If the Eyes Diſtance being given, ſuch a Diſtance between the *Object* and the *Table* were required, that DLFEZT : BIGCOV : : p : q. Since p : q : : DLFEZT : BIGCOV : : $EF^q : GC^q$ (becauſe the Figures are Similar) : : $AE^q : AC^q$ (Similar △^ls) : : $AQ^q : AP^q$ (Similar △^ls.)

'Tis plain, that supposing the Line APQ perpendicular to the two Planes, the Distance PQ is easily determined; *viz.*

$$PQ = \frac{\sqrt{p} - \sqrt{q}}{\sqrt{q}} \times AP.$$

COROLL. III.

The Object continuing in a Position parallel to the Table; whether the Eye moves nearer or farther from the Table, while the Object keeps its Distance, or the Object moves while the Eye keeps the same Distance, or the Table moves, while the Eye and Object keep their Places; in either Case, there is no Alteration, of the *Species*, of the Perspective, but only of the *Magnitude* thereof. But of these things, in another place.

PROP. VI.

All the Conical Sections, *are only the Perspective Representations of the* Circular *Line, of the* Base, *upon Tables in various Positions, to the Eye seated in the* Vertex *of the Cone.*

It will need no Figure, to prove this Proposition to them, that know the Cone, and the several Sections of it.

 For

For the Cone being cut by a Plane, parallel to a Plane, which coming out of the Vertex, touches the Cone in its Side; (or which is all one, meets the circular Base in *one Point* only) if the Plane of this Section, be made the Perspective Table, the Representation of the Circular Arch, will be a *Parabolical* Line.

But if the Table be parallel to a Plane, which meets not the circular Base, at all; it will be an *Ellipsis*; or if parallel to a Plane, which cuts the Basis, an *Hyperbola*.

The Reason is, because the right Lines, drawn on the Surface of the Cone, from the *Vertex*, to the several Points of the Circumference of the Base (which Lines in this Case, are our *visual projecting Rays*) do trace out upon the Planes of the several Sections (which are our *Perspective Tables*) the Conical Curves; which therefore are only so many Peices of Perspective, to an Eye posited in the Vertex. *Q. E. D.*

COROL. I.

One and the same Conick Section, may be the Perspective of an infinite Number of different circular Arches. For the *Geometricians* demonstrate, that any *Parabola*, may be adapted to *any Cone*; and any *Ellipsis* or *Hyperbola* (though not to any Cone, yet

least

leaſt) to various Cones, *differing in Species.*

COROL. II.

Any of the Conick Sections may be, the Perſpectives of each other, to the Eye (as before) plac'd in the Vertex.

Ex. Gr. Suppoſe a Plane cutting the Cone, and producing a Parabola. Thro' the common Section of this Plane with the circular Baſe, ſuppoſe an infinite Number of other Planes to paſs, each cutting the Cone between its Vertex, and the Vertex of the forementon'd Parabola. Any one of this infinite Number of Planes, being taken at Liberty, for a *Table*; the Eye ſees the *Parabola*, as an *Hyperbola* thereon.

And ſo of any of the reſt. The thing is ſo plain, that any one, by only drawing a Cone, may abundantly ſatiſfie himſelf of all the Particulars.

SCHOL. I.

From *this Generation of the Conick Sections*, wherein we conſider them as the Perſpectives of the circular Baſe, ariſes a Speculation, which is not unworthy of Notice; and that is this.

The whole Area of the *Ellipſis* in any Cone, lying all entirely *above* the Circular

 Baſis,

Basis, or between the Vertex and it, is therefore, the Projection of that whole Basis.

But the *Hyperbola and Parabola*, being Curves which do not *include Space*, but run out *ad infinitum*, are projected after another Manner. In the *Parabola*, *ex. gr.* that Part which *lies above* the circular Basis, is the Projection of a *determinate Arch* of the Circle, and the remaining *infinite* Portion thereof *below* the Basis is projected from the *Complement* (of the said Arch) to the whole Circle. For the *last* projecting Rays is the Side of the Cone, parallel to the Axis of the Parabola.

In the *Hyperbola*, that Part which is *above* the Basis of the Cone, is likewise the Projection of a *determinate* Arch of the Circle, but the remaining *Infinite* Portion *below* the Basis, is projected, not from the Complement of the former Arch, to the whole Circle, but from the Complement thereof, *to that Arch, which is determin'd, by a Plane passing out of the Cones Vertex, parallel to that which generates the Hyperbola.* I say, the infinite Portion of the Hyperbola below the Basis of the Cone, is form'd, by projecting only that Arch (which lies between the Plane *making the Section*, and the Plane out *of the Vertex* parallel thereto) upon the Plane making the Section.

Now,

Now, as in all Projection whatſcever, either the Plane we project on, is plac'd between the Object and the Eye, or elſe the Object, between that Plane, and the Eye; ſo it has been uſual to call the latter Sort of Projection, an *INVERTED PERSPECTIVE, or a DEFORMATION:* For in all the common *Scenographick* Repreſentations, the Table is always plac'd between the Eye and the Object. *Now both theſe kinds of Projection take place, in that Generation of the Conick Sections, we are ſpeaking of.* Nay, and both too, in the Formation of one and the ſame Section.

The *whole Ellipſis*, is a regular *Scenographick* Projection, the Table being between the Object and the Eye.

So likewiſe are thoſe Portions of all *Parabolas and Hyperbolas*, which lie above the Baſis of the Cone.

But the remaining *Infinite* Portions of thoſe Curves, *below* the Baſis, are *Inverted Perſpectives* or *Deformations*; the Circular Arch, which is the Object projected, lying between the Eye, and thoſe Parts of the Planes of theſe Sections, on which the Projection is made.

SCHOL. II.

Since the ſame general Affections which are demonſtrated of Cones, whoſe Baſes

are *Circles*, are applicable likewise to such Cones, whose Bases are *any of the Conick Sections*, (*per Append. de Sectionibus Pyramidum quarum Bases sunt Sectiones Conicæ*; *M. de La Hire*) 'tis plain from hence, that we may determine how, and in what Circumstances, any Conick Section, *seated in the Ground Plane*, shall become any other Conick Section whatsoever *in Perspective*: That is, what Section shall be produced, by the Plane of the Table cutting any sort of Cone, whose Base is either *Ellipsis*, *Parabola*, or *Hyperbola*.

PROP. VII. (Fig. 7.)

It may be, that Lines, which are not parallel in the Ground Plane, *may come into parallel Lines on the* Table. *Or*, *The Perspectives of* Diverging *Lines*, *may be* Parallel.

Suppose the Non-parallel Lines to be PD, NE, the Eye at K, its Height KV, the Table, RSCT.

Let the Eye be so posited, that the Lines PD, *NE*, *may lie in the visual Planes* KVD, KVE; whose Intersections with the Table are AP and BN, and therefore the Representations of the aforesaid Lines to the Eye at K.

I say that AP is parallel to BN, if the Eye be so placed. For

For becauſe KV is perpendicular to the Ground Plane, therefore the Planes KVD, KVE, are perpendicular to the Ground Plane. And becauſe the Table RSCT is likewiſe Perpendicular thereto; therefore AP, and BN, the common Sections of theſe Planes, are perpendicular to the Ground Plane, and therefore parallel to one another. *Q. E. D.*

COROLLARY I.

The Trapezium PDNE is repreſented on the Table, by the Rectangle APBN.

COROL II.

Hence the vulgar Method of rectifying a deformed Object; or placing the Eye in ſuch Manner, that a *rude and irregular Picture*, ſhall from a certain Point, appear regular and beautiful. For thus, the Trapezium PDNE, which may be as diſtorted and unſhapen as one pleaſes, will fall on the PerſpectiveTable in the compact Form of a Rectangle, as APBN. And therefore were the Parts of any Image, ſuppoſe a *Humane* Face) diſpoſed up and down in the *Cells* of this Trapezium, they would appear, in an agreeable Order and Poſture to the Eye, in the correſpondent Cells of the Rectangle upon the Table.

COROL.

COROL III.

Hence it appears, that this Practice of *Deforming*, is rightly Term'd, *An Inverted ſort of Perſpective.*

For as in the *common* Perſpective, a Rectangle *ex. gr.* lying in the Ground or *Horizontal* Plane, is projected into a Trapezium upon a *vertical* Table, placed between the Object and the Eye; ſo in *Deformations*, a Rectangle drawn in a *vertical* Plane, is projected into a Trapezium, upon a *Horizontal* Table, which lies farther from the Eye than the Object does.

COROL. IV.

The Points D, and E, and conſequently the whole Deformation, are determin'd, by drawing out the viſual Rays KA, KB, till they interſect the Ground Plane in D, E, and then joyning DE.

COROL. V.

Otherwiſe, the Lines VP, KA, and VN, KB, produced till they meet each other; meet in the ſame Points D and E, as before.

COROL.

COROL. VI.

The Line DE is parallel to PN, ſo that the Deformation of a Rectangle, is a Trapezium, whoſe two oppoſite Sides are parallel.

PROP. VIII. (Fig. 7.)

There is an infinite Number *of* Points, *in which the Eye being placed, ſhall project* Diverging *Lines upon the* Ground Plane, *into Parallel ones on the* Table; *and the* Locus *of thoſe Points, is eaſily determin'd.*

Suppoſing all, as in the foregoing Propoſition: Produce the diverging Lines DP, NE, till they cut each other in V; at which Point, erect the Perpendicular VK, which extend at Liberty. I ſay, the Perſpectives of the Lines DP, EN, ſhall be parallel to each other on the Table, the Eye being placed in any Point of this *infinite* Perpendicular.

For taking any Point therein, as K, for the Eye's Place, and drawing out the *viſual* Planes KPD, KNE; it's evident that theſe coincide with the Planes KVD, KVE which being perpendicular to the *Ground* Plane VDE, their common Sections with the

the Plane of the Table, AP, BN, will be demonſtrated (as before) to be parallel to each other. Therefore, *&c.* *Q. E. D.*

SCHOL.

We have in this Caſe, an Inſtance (tho' depending on another Principle) of the Truth of what was before diſcours'd at *Schol. Prop.* 3. viz. *Concerning Lines which are not parallel, appearing as Parallels.*

For ſince all the Points of the Lines PD, NE, appear in AP, BN, which are ſtrictly parallel to one another; 'tis evident that the two *former* Lines are ſeen *as* the two *latter*, that is, *as* parallel Lines. And how that is, we have demonſtrated at Prop. 1, 2, 3.

LEM. I. Fig. 8.

If the Parallels LM, GV, HN, *&c.* in the Baſe of the Triangular Priſm ABLHMN, be produced at Liberty towards P, Q, R, *&c.* any Lines as AI, AK, drawn from the ſolid Angle A, to any Points as I, K, in thoſe Parallels, ſhall neceſſarily interſect the Lines NB, VB, running up from the Points N, V, to the other ſolid Angle B.

DEMON.

DEMONSTRATION.

For the Line NR, *ex. gr.* being in the Plane of the Parallelogram ABHN; 'tis evident, that a Line drawn from A to a Point as I, in the Base HN, produced, shall cut the opposite Side BN in some Point as *e*, by the way; so in the Parallelogram ABGV, the Line AK, shall cut BV, in *k*; and so of all the rest. Therefore, *&c.* *Q. E. D.*

LEM. II.

If an Eye were plac'd at A, it would project the Point I into *e*, or K into *k*; or any other Points in the parallel Lines RN, VQ, into some Points of the Lines NO, VS, running up to the Angle B. And therefore, it would project the parallel Lines NR, VQ, MP, infinitely produced, into the Lines NB, VB, MB.

PROP. IX. Fig. 8.

The Perspectives, of all Lines which are parallel one to another, and not parallel to the ground Line, do run up into one and the same Point in the Table.

This is the main and great Proposition in this Science, and is thus easily and universally

versally demonstrated, by the Help of the two foregoing Lemmata.

DEMONSRATION.

Suppose the Parallels MP, VQ, NR, *&c.* I say, the Perspectives of these shall all run up into one and the same Point. By Lemma 1. the common Sections of all the Planes, AHR, AGQ, ALP, *&c.* with the Plane CDFE, must necessarily meet one another in the solid Angle of the Prism B.

By Lem. 2. the Plane CBDFVE is a Perspective Table to the Eye plac'd at A, the other *solid* Angle of the *same* Prism; and therefore the common Sections of the aforesaid Planes, AHR, AGQ, ALP, *&c.* with the Plane CDFE, will be the Perspectives of the Parallels NR, VQ, MP, and therefore these Perspectives must necessarily be the Lines NB, VB, MB, *&c.* all meeting in one and the same Point B. *Q. E. D.*

COROL. I.

From hence again appears a Reason why in long *Rooms* and *Walls*, or Rows of *Trees* and *Pillars*, the Sides seem closer one to another towards the *farther* End, then at the Parts nearer the Place of the Eye.

COROL.

COROL. II.

If any Object, as I, were removed out to the Horizon, the visual Ray AI, would coincide with ABO, so would all other visual Rays coincide with each other in the same Ray AB.

COROL. III.

From hence it follows, that the Point to which the Perspectives of any Parallels converge, is there where a Line from the Eye parallel to those Parallels, strikes the Table. For all the visual Rayes coincide at last in the Ray AB; which is the *common Section* or Side, of all the parallelogram Superficies ALOP, AGOQ, AHOR, *&c.*

Therefore since (from the Construction of the Figure) AB is parallel to each of the infinite Lines in the Base; the Consequence is clear.

COROL. IV.

The Ray AB strikes the Table at right Angles, when the Lines NP, NR, *&c.* are perpendicular to the ground Line MN. But if MB, NR, *&c.* cut the ground Line obliquely; the Angle ABC will still equal

equal PMN or RNE, *&c.* that is the Ray AB, will always make the ſame Angle with the Line CBD in the Table (which we ſuppoſe parallel to MN) as the oblique Parallels themſelves, do with the Line MN.

COROL. V.

If the Plane MPNR be parallel to the *Horizon*; 'tis plain that the Line CBD, will be that, which we call the *Horizontal* Line; whoſe Elevation above the ground Plane, is juſt equal to the Height of the Eye. And conſequently, it will follow, that the Perſpectives of all Parallels, whether *perpendicular* or *oblique* to the ground Line, do run up to ſome Points in the *Horizontal* Line. But if the Plane MPNR, be either *elevated* above, or *depreſs'd* below, the Horizontal Plane; then the Point where Perſpectives of theſe Parallels will meet, will accordingly be found in the Table *above* or *below* that, which is commonly called the *Horizontal Line*.

COROL. VI.

If the Parallels MP, VQ, NR, *&c.* be at right Angles to the Line FE, then B, ſhall be that, which we call the *Point of Sight*;

Sight; but if the ſaid Parallels be oblique to the Line FE, then B ſhall be ſome *Accidental Point*.

COROL. VII.

From what has been ſaid, appears the Method of finding out the Points, to which the Perſpectives of any Parallels, lying in the Plane of the Horizon, do converge upon the Table. *Viz.*

Draw a Line from the Foot of the Eyes Perpendicular, parallel to the Parallels propos'd, and ſee where it cuts the Ground Line. From that Point carry up a Perpendicular, equal to the Height of the Eye. Where that Perpendicular interſects the Horizontal Line, will be the Point ſought.

Therefore, to determine the Point of Sight, is only to let fall a Line from the Eye, *perpendicularly* to the Table.

COROL. VIII.

When the oblique Parallels, cut the Ground Line, at an Angle of 45°; then the *Points of Diſtance*, become the proper *Accidental* Points, to which the Perſpectives of of thoſe Parallels converge.

COROL. IX.

By how much the more obliquely, any Parallels fall on the Ground Line, by so much the farther, is the Point to which their Perſpectives converge, diſtant in the *Horizontal* Line, from the *Point of Sight*.

COROL. X.

If any Angle be made *at the Eye*, equal to the Angle contained *under the Sides* of any Poligon; the Leggs comprehending that Angle, will ſtrike the Table in thoſe Points, to which the Perſpectives of all Lines, parallel to the ſaid Sides, will converge. Thus in an *Equilateral* Triangle; *ex. gr.* Thoſe Points will be determined by the Leggs of an Angle of 60°: In a *Square*, by an Angle of 90°: In a regular Pentagon, by one of 108°; and ſo in every *regular* Figure the *accidental Points* (to which the Perſpectives of all Lines parallel to the Sides of that Figure, converge) are marked out by Rayes, making an Angle at the Eye, equal to the Angle of the ſaid Poligon.

COROL.

COROL. XI.

The Perſpectives of all oblique Paral-les in the Horizontal Plane both *below*, and *bove* the Eye, and in the two ſide Planes, erpendicular to the former, will all con-ur in one and the ſame Point. So that n infinite *Parallelippid*, dwindles in Per-ſpective, into a *Pyramid*, as a *Parallelo-ram* does into a *Triangle*, and a *Cylin-er* into a *Cone*.

COROL. XII.

If the Right Line, which is the Eyes iſtance from the Table, be produced in-initely towards the Parts of the Eye; the ame converging Lines on the *Table*, will e the Perſpectives of the ſame Parallels n the *Ground Plane*, to the Eye ſeated in ny Point whatſoever, of that infinite ine.

And this ſolves, what ſome have reckon'd ſort of *Paradox*, in this Science, *viz*. That he *ſame Parallels ſhould be projected in-o the ſelf ſame Lines on the Table, tho' the Eye changes its Place and Diſtance.*

SCHOL.

Since the Base of the Prism (Fig. 8.) may be as well *any sort* of Parallelogram, as a *Rectangle*; as also since the two opposite Triangular Planes, may be as well any way *inclin'd*, as stand *perpendicular* to the Plane of the Base: It follows, that the Proposition, is by *this Method* universally demonstrated, with Respect to any sort of Lines, drawn in Planes, which lie in any Manner of Position to the Table. For of what Species soever the Prism be, provided it be but a Prism, yet still the Lines MB, VB, NB, which run up to one solid Angle B, will necessarily be the Projections of the Lines MP, VQ, NR, to the Eye placed at the other solid Angle A.

PROP. X.

The Perspective of any visible Point, *is truly determin'd, by the Intersection of a* Radial *Line, (drawn from the Point of* Incidence) *and a Line connecting the Eye's Distance, set off in the* Horizontal *Line, with the Distance of the Point seen laid off in the* Ground Liue. (See Fig. 9.

CON-

CONSTRUCTION.

Let C b n d be the Table, the Eye at A, its Height AG = BH, the Diſtance of the Point ſeen, D from the Ground Line = ID = IE, the Diſtance of the Eye GH = AB = BC; the Radial BI drawn from the Point of Incidence I, to the Point B; the Line CE connecting the Points C, E, cutting the Radial BI in L; the Line AD, drawn from A to the Point ſeen D, cutting the Table in the Point K.

I ſay that L is the Perſpective of D.

Draw CM parallel to BI, and FI = and parallel to GH; Join the Points GF, and laſtly draw AF.

DEMONSTRATION.

Since the viſual Ray AD, cuts the Table in the Point K, ; 'tis plain from thence that K is the true *Natural* Perſpective of the Point D.

Alſo by Propoſition IX, it appears, that *K* the Perſpective of *D*, muſt needs be found in the *Radial* BI, drawn to the Point of *Sight* B, from the Point of *Incidence* I, I ſhall now demonſtrate, that the Point L coincides with *K*, the natural Perſpective of the Point D.

The $\triangle^{ls}$ DAF and DIK are Similar, for AF ||[l] BI.
Therefore DF : AF :: DI : IK.
Alſo the $\triangle^{ls}$ CME and LIE are Similar, for CM || BI.
Therefore ME : MC :: IE : IL.

Now becauſe EI = DI, therefore DI + IF = IE + IF = IE + GH = IE + AB = IE + AB = IE + CB = IE + MI, therefore DF = ME.

Again, AF = BI = MC. So that the three firſt Terms of the *firſt* Proportion, are reſpectively equal to the three firſt of the *latter*, therefore the fourth Terms are reſpectively equal, that is IK = IL. Therefore the Points K and L coincide. Therefore the Perſpective of the Point D, is truly determin'd by the Interſection of the Line CE, with the Radial BI. *Q. E. D.*

COROL.

Hence it follows, that the Perſpective of any Point, will alſo be determin'd, by the mutual Interſection, of the Lines drawn from the *two Points of Diſtance*, to thoſe Points in the *Ground Line*, where the Diſtance of the Point ſeen, is laid off. That is, ſet off the *Eyes Diſtance*, both ways, from the *Point of Sight*, in the Horizontal Line;

Line; and the *Diſtance of the viſible Point* from the Table, both ways, from the *Point of Incidence*, in the *Ground Line*, and connect the Points above and below *Alternately*, with right Lines; ſo ſhall the Interſection of theſe two Lines in the Table, be the true Perſpective of the Point given.

For it may be demonſtrated by the ſame Steps, as above, that each of theſe Lines of *Diſtance*, will interſect the *common Radial* (drawn from the Point of Incidence) in one and the ſame Point. Therefore, *&c.*

PROP. XI.

Any Portion of a direct *Line*, contiguous *to the Table, is to its Perſpective, as the Sum of its Length, and the Eyes Diſtance from the Table, is to the Length of the whole correſpondent Radial.*

CONSTRUCTION.

Let the Diſtance propoſed be TM. (See Fig. 10.) The Perſpective of the Point M is at E, ſome where in the Radial TF, drawn from T the Point of Incidence, by Prop. 5. Draw GY parallel to AB, and produce MT to cut GY in V. Then draw KV, from the Eye at K.

DEMONSTRATION.

Becauſe GY || AB, and VT || Gs, therefore GV = ST, and ſince KG = FS, and the Angle KGS = FST, being both ∟rs, therefore FT = KV. Farther ſince VT = GS, = KF, therefore VT = KF. Therefore the Figure KFVT is a Parallelogram, therefore FT || KV. Therefore the △ls KVM and ETM are Similar;

Therefore TM : TE : : VM : VK, but VK = TF,

Therefore TM : TE : : VM : TF. *Q. E.D.*

COROL. I.

When the Diſtance TM, coincides with SO, which runs up to the Foot of the Eyes perpendicular; then the Rule will be thus: *As the Diſtance ſeen, is to its Perſpective, ſo is the Sum of that Diſtance, and the Eye's Diſtance from the Table, to the Height of the Eye.* For now the Length of the *Radial*, coincides with the Height of the Eye.

COROL.

COROL. II.

Equal Portions being taken, of ſeveral *direct* Lines; that which paſſes thro' the Foot of the Eyes perpendicular, will have its perſpective Contraction, of all the *ſhorteſt*.

COROL. III.

Hence may be computed the Proportion, between the Perſpectives of any Part of a *direct* Line contiguous to the Table, to the Eye plac'd, either at different *Heights* or different *Diſtances*, or different Heights and Diſtances both together; *viz.* By *Corol.* I. when the Line paſſes through the Foot of the Eye's Perpendicular; and by the *Prop.* it ſelf, when it paſſes through any other Point.

COROL. IV.

And becauſe the Perſpective of any Part of a direct Line, not *contiguous* to the Table, is equal to the Difference of the Perſpectives of two Parts of the ſame direct Line, *which are contiguous* to the Table; therefore, by what has been ſaid, we can alſo determine the Proportion of the Perſpectives,

ſpectives, of any Segment of a direct Line *not contiguous* to the Table, to the Eye either at different *Heights*, or *Diſtances*, or both together. But of this, ſee more by and by, at Prop. XIII.

PROPOSITION XII.

If any Line be parallel *to the Ground Line, its Perſpective in the Table, ſhall be parallel to the* Ground Line *alſo.*

CONSTRUCTION.

Let the Line MN be parallel AB, (See Fig. 10.) the Ground Line; and drawing from the Eye at K, the Lines KN, KM, let us conceive the *Plane of Rayes* KNM, whoſe common Section with the *Plane of the Table*, ſuppoſe to be DE, which is therefore the Perſpective of MN (by Def. XX.) and muſt now be ſhewn to be parallel to AB. Upon MN, erect the Plane MNXZ, perpendicular to the Ground Plane.

DEMONSTRATION.

Becauſe the Plane NMXZ is perpendicular to the Ground Plane, thereſore it is parallel to the Plane of the Table. And becauſe the Plane KMN, cuts the Table and

this this Plane NMXZ, therefore the common Sections shall be parallel.

But these common Sections are MN and DE. Therefore MN and DE are parallel; but MN is parallel to AB (by the *Hypothesis*) therefore DE is also parallel to AB. *Q. E. D.*

COROL. I.

Therefore if MN and HL are two Lines parallel to the Ground Line, their Perspectives DE and PR, shall be parallel to one another in the Table.

COROL. II.

If any Line NM parallel to the ground Line, be bisected in O, a Ray drawn from the Eye to the Point of Bisection, shall bisect the Perspective of the said Line DE, in C,

For the Triangles KCE and KOM are Similar. Therefore OM : CE : : KO: KC.

Again, the Triangles KON and KCD are Similar.

Therefore ON : DC : : KO : KC.
Therefore OM : CE : : ON : DC.
But OM=ON, therefore CE=DC.

COROL.

COROLL. III.

The ſame things being ſuppoſed as before, I ſay the Line NM is to its Perſpective, as the Diſtance between the Foot of the Eye's Perpendicular and the Line NM, to the Diſtance of the Eye from the Table.

For the $\triangle^{les}$ KDE and KNM are Similar, therefore NM : DE :: KO : KC.

Again, the $\triangle^{les}$ KGO and CSO are Similar;

Therefore KO : KC :: GO : GS.
Therefore NM : DE :: GO : GS, but KF = GS,
Therefore NM : DE :: GO : KF. *Q. E. D.*

PROP. XIII.

The Perſpectives of all Lines perpendicular *to the ground Plane, will, if produced in the Table, be perpendicular to the* ground Line.

CONSTRUCT. (Fig. 11.)

Let CD be a Perpendicular to the ground Plane; and let the *erect* Plane RCSDT, paſſing thro' the Line CD, be parallel to the Table. From the Points C, D,

C, D, draw AC, AD, to the Eye at A. And let the Triangular Plane of Rayes, ACD, make EM for its common Section with the Plane of the Table.

DEMONSRATION.

Becauſe CD and EM, are the common Sections of *two* parallel Planes by a *third* Plane, they ſhall be parallel to one another: Therefore EM if produced, ſhall be perpendicular to the ground Line HLP. The ſame may be demonſtrated of NO the Perſpective of IK. Therefore, *&c.* *Q. E. D.*

COROL. I.

Hence the Perſpectives of all Perpendiculars to the ground Plane, are parallel one to another in the Table.

COROL. II.

The Perpendicular CD, is to its Perſpective EM, as the Sum of the Diſtances of the Eye, and of that Perpendicular from the Table, to the Diſtance of the Eye from the Table; or, as the Diſtance between the Foot of the Eyes Perpendicular, and the Perpendicular ſeen, to the Diſtance of the Eye from the Table.

For

For from Similar $\triangle^{ls}$ ACD and AEM, 'tis DC : ME : : AD : AM.

And from Similar $\triangle^{ls}$ ADF and MDL, it is AD : AM : : FD : FL.

Therefore DC : ME : : FD : FL. *Q. E. D.*

COROL. III.

If two or more Perpendiculars to the Ground Plane, which are of *equal Height*, do alſo ſtand at equal Diſtances from the Table; their Perſpectives ſhall be equal.

Let the Perpendicular DC = Perpendicular IK : and the Diſtance LD = IH.

And let ME be the Perſpective of CD, and NO the Perſpective of KI. It was ſhewn in the Propoſition, that CD : ME : : FD : : LD, therefore for the ſame Reaſon, IK : NO : : GI : HI. But HI = LD (by the Hypotheſis) and ſince FL = AB = GH, therefore GI = FD, therefore CD : ME : : IK : NO; but CD = IK, therefore EM = NO. *Q. E. D.*

COROL. IV.

Any *Perpendicular to the ground Plane*, is to its Perſpective; as a *Parallel to the ground Line*, at the ſame Diſtance from the Table, is to its Perſpective. Becauſe they are on both Sides proportional to the Lines

FD

FD and FL; as appears by comparing Cor. II. of this Propoſition, with Cor. III. of Propoſition XII.

And therefore, if the Perpendicular and the Parallel, are equal in the Length, their Perſpectives will be equal alſo.

COROL. V.

The *Diſtance* of the *Object* and *Eye* from the *Table* continuing; the Perſpectives of the *ſame* Perpendicular, are equal, whether the Eye be plac'd at a *leſs* or a *greater* Height.

The *Object* PG (Fig. 11) the *Table* TK, the *Eye* at the two different Elevations A and S, in the ſame Perpendicular AC. Draw the Rayes AP, AG, SP, SG, interſecting the Table in the Points q, o, p, e.

Now, PG : qo :: AG : Ao (Similar $\triangle^{ls}$, APG, Aqo) : : CG : CK (becauſe AC parallel TK).

Again, PG : pe : : SG : Se (Similar $\triangle^{ls}$, SPG, Spe) : : CG : CK (becauſe of the ſame Parallels.)

Therefore PG : qo : : PG : pe, therefore qo = pe. *Q. E. D.*

COROL.

COROL. VI.

The *Height* of the Eye continuing, as alſo the *Diſtance* between the Object and Table: The Perſpectives of the ſame Perpendicular, to the Eye at *two ſeveral Diſtances*, are in the Ratio compounded, of the *direct* Ratio of the Diſtances of the Eye from the Table, and the *Reciprocal*, of the Diſtances between the Object and the Eye.

The Object PG, as before, its Perſpectives to the Eye at A and D, qo, and gb, reſpectively; determin'd by the Interſections of the Rayes AP, AG, DP, DG, with the Table TK.

'Tis, PG : qo :: AG : AO (Similar $\triangle^{ls}$, APG, Aqo)

AG : Ao :: CG : CK,

Therefore PG : qo :: CG : CK.

Again, PG : gb :: DG : Db (Similar $\triangle^{ls}$, DPG, Dbg)

DG : Db :: EG : EK,

Therefore PG : gb :: EG : EK.

$$\because qo : gb :: \frac{PG \times CK}{CG} : \frac{PG \times EK}{EG} ::$$

$$\frac{CK}{CG} : \frac{EK}{EG} :: CK \times EG : EK \times CG.$$

Q. E. D.

From

From hence it follows, that the Perſpective, to the *remoter* Eye, is greater than that to the *nearer*. For ſince $EG = EK + KG$, and $CG = CK + KG$, therefore $qo : gb :: CK \times EK + CK \times KG : EK \times CK + EK \times KG$; but $EK > CK$, therefore $EK \times KG > CK \times KG$, $\therefore EK \times KG + EK \times CK > CK \times KG + EK \times CK$, $\therefore gb > qo$. *Q. E. D.*

S C H O L.

By what has been demonſtrated at Prop. II. and (with their reſpective *Corollaries*) may be determin'd, whatever relates to the Proportion between the *Perſpectives* of any *direct* Lines, and of any Lines *Perpendicular to the ground Plane*; whether they be *contiguous* to the ground Line, and to the ground Plane, or *not contiguous*, and that for all the various Caſes, of different *Heights and Diſtances* of the Eye. 'Twould be too tedious a Work, to go through them all here. 'Tis ſufficient to have ſhewn the way, both by *Precepts and Examples*, how to proceed in any of them that may occur. But for a farther Help, *this preſent Figure* may be conſidered, wherein the Eye is plac'd at ſeveral Elevations and Diſtances from the Table, and the Perſpectives of *both Sorts* of Lines mention'd, are diſtinctly

represented, to put the Reader, the more easily, into the Way of arguing out any of these Particulars. Thus *eg. gr.* the Perspective of the *direct* Line GH, not contiguous to the Table, is the same with the Perspective of the Line GP, perpendicular to the Ground Plane, the Eye being, in either Case at A. And so of the rest.

PROP. XIV.

If any direct *Line be divided into any Number of* equal *Parts, the Perspectives of those equal Parts shall be* unequal.

CONSTRUCT. (See *Fig.* 12.)

Let the direct Line be DF, the two equal Parts BG, GF, their Perspectives NL, and LM, determin'd by the Intersection of the Rayes AB, AG, AF, with the Line SD, drawn from the Point of *Incidence* D, to the Point of *Sight* S. From the Point N, draw NO, parallel to the Line DF, and from the Point C, draw RC, parallel to AF.

DEMONSTRATION.

The $\triangle^{ls}$ ABG and ANC, AGF, ACO are Similar, therefore AG : AC :: BG : NC.

Also

Also for the same Reason, $AG : AC :: GF : CO$, therefore $BG : GF :: NC : CO$, but $BG = GF$, therefore $NC = CO$. Again, the $\triangle^{ls}$ NMO, NRC are Similar, therefore $NC : CO :: NR : RM$, therefore $NR = RM$.

In the $\triangle^{l}$ AGB, the $\angle^{l}$ AGB is $>$ than the $\angle^{l}$ AFB, but $MON = AFB$, and $LCN = AGB$, therefore $LCN > MON$. Now RC, parallel MO, therefore $RCN = MON$, therefore $LCN > RCN$, therefore $LN > RN$. But $RN = RM$, therefore $LN > RM$, therefore $NL > LM$.

Q. E. D.

COROL.

Hence it follows, that the Divisions of any *Radial* Line, in the Perspective Table, which answer to any *equal* Divisions of a *direct* Line; are not only *unequal:* But also that the Parts grow less and less, as they approach nearer in that *Radial*, to the *Point of Sight*.

PROP. XV.

If a Line be inclined, by any Angle whatsoever, to a Transverse *Line in the* ground Plane; *its Perspective shall make the same Angle with the Perspective of that*

Tranſverſe *Line in the Table.* (See Fig. 13.)

CONSTRUCTION.

Let the inclin'd Line be AC making any $\angle^{l}$ as ACB, with the Tranſverſe Line BC; from any Point as A, in the Line AC, let fall the Perpendicular AB. From the Eye at R, draw the Rayes AR, RC, RB, interſecting the Table in the Points *a*, *b*, *c*, which are therefore the Perſpectives of A, B, C.

DEMONSTRATION.

Since BC is parallel to the ground Line NS (by Hypoth.) therefore *bc* parallel NS by Prop. XII. Farther, ſince AB perpendicular BC, therefore alſo by Prop. XIII. *ab* will be perpendicular *bc*; ſo that the $\triangle^{ls}$ ABC, *abc* are Rectangular at B, and *b*. Now by Corol. IV. Pro. XIII. it appears that AB : BC :: *ab* : *bc*, becauſe both are in Proportion of FB : Fn. Therefore the $\triangle^{ls}$ ABC and *abc* are Similar, having their Sides about the equal Angles proportional. Therefore the $\angle^{le}$ *abc*$=\angle^{le}$ ABC. *Q. E. D.*

PROP.

PROP. XVI.

If in the ground Plane, there be drawn any Number of Parallels *to the* ground Line, *being all of the same* Bigness; *and at the Extremities of them be erected Perpendiculars to the ground Plane all of the same* Height; *the Perspectives of these* Parallels *and* Perpendiculars, *shall divide all the* Radials, *drawn through the Extremities of the said Perspectives, in the same Proportion.*

CONSTRUCT. (Fig. 14.)

Let the Parallels to the *ground Line*, be OH, MI, NK, included between the same Parallels EN, FK, and the respective Perpendiculars HR, QI, PK, whose Tops are terminated in the Line RP, parallel to FK. Produce the Line RP till it cuts the Table in *d*; and draw the Radial Cd. Let S, V, W, be the Perspectives of N, M, O, and m, n, o, those of K, I, H, and X, Y, Z those of P, Q, R, Lastly, draw the Radials CSE, CmF, CXd.

DEMONSTRAT.

By Prop. XII. Cor. I. Sm, VN, Wo, are Parallels. Therefore SV : VW :: mn : no. Again, by Prop. XIII. Cor. I. Xm, Yn, Zo, are Parallels, therefore mn : no :: XY : YZ; therefore SV : VW :: XY : YZ. Therefore all the *Radials* are cut proportionally. *Q. E. D.*

The Practice of PERSPECTIVE.

The Practice of Perspective is Twofold, *Direct* and *Inverse*.

The *direct* Method, is that, by which we trace out the Appearance of any *given Object*, upon the Plane of the Table.

The *Inverse*, that, by which from the *Perspective given*, we go back to the Object it self; and so by a sort of *Linear Investigation*, shew the Work to be rightly done. This is very useful and necessary in some Cases, where a Doubt may arise concerning the Exactness of an Operation; and in any Case, 'tis very pleasant, thus to bring what is done to a regular Examen.

The *Geometricians* have their *Synthesis* and *Analysis*, or *Compositive* and *Resolutive* Methods; and the *Analysts*, their *Direct* and *Inverse* Method of Fluxions. And as 'tis

'tis a ſure Proof, that a *Fluent* is rightly determin'd, when the Fluxion thereof is exactly equal, to the Fluxion at firſt propoſed; ſo 'tis certain likewiſe, that the Work is right in *Perſpective*, when by a fair Proceſs from what is done, we can come back to the true *original* Object it ſelf. And the *Parallel* would be exact in all Reſpects, if we did but ſhift Names, and call that the *Inverſe*, which before we call'd the *Direct* Part of the Practice of Perſpective: We ſhall exemplifie *both* theſe Branches diſtinctly. But to proceed.

Hitherto we have repreſented the Perſpective Table as ſtanding *ſideways*, with Reſpect to the Eye of the Perſon that looks on the Page, where the Figures are drawn. And 'tis certain that this ſerves, to give the cleareſt Idea, of the Demonſtrations, of the Propoſitions, that are advanced in this Science. But now it will be neceſſary to repreſent the Table after another manner; that is *foreright* or *direct* to the Eye of the Reader: So that whereas, before, it was imagin'd to ſtand at *Right Angles* to the Plane of the Page, now we are to conceive it as *lying in that Plane*; this ſort of Repreſentation, being the moſt commodious for Operation and Practice.

I ſhall begin with the *firſt Branch* of the Practice of Perſpective, *viz. How to delineate the true Scenographical Appearance of any Object given*; or how to proceed from, the Object to the Perſpective; which is the *direct* Method.

And in order to the making all things here as eaſie as may be, I premiſe, that the Method of *determining the Perſpective of a Point*, being ſhewn at Prop. X, and the Corollary thereof; the Perſpectives of all *Lines* and *Figures*, are from thence likewiſe determinable.

The Perſpective of a *Right Line*, is had, by finding, and joining the Perſpectives, of its *Extremities*.

The Perſpective of a *Rectilineal* Plane Figure, is had by determining the Perſpectives of all its *Sides*.

The Perſpectives of *Crooked* Lines, or *Crooked-lin'd* Plane Figures, are determin'd (at leaſt exactly enough for Practice) by carrying a Crooked Line, thro' the Perſpectives of a ſufficient Number of Points.

The Perſpective of a *Solid*, whether *Rectilineal* or *Curvilineal*, is determin'd by finding firſt, the Perſpective of the *Baſe*, and then ſetting off the Perſpectives of the *Heights*, from their proper Points of Seat in the Baſe, and joining the Extremities.

To

To go on a little farther, with theſe *General Directions*; let it be obſerv'd, that the Perſpectives of all Lines, which cut the Ground Line, at *Right Angles* are to be carry'd up to the *Point of Sight*; or if at oblique Angles, then to ſome *Accidental* Point, determin'd according to *Corol.* VI. and *Schol. Prop.* IX.

And in General, that the Perſpectives of all Lines parallel to each other, do run up into one and the ſame Point, in the Table; by the aforeſaid *Prop.* IX.

That the Perſpectives, of all Lines *parallel* to the Section or Ground Line, are to be drawn parallel to it, upon the Table; by *Prop.* XII.

That the Perſpectives of all Lines *perpendicular* to the Ground Plane, are to be drawn in the Table, perpendicular to the Ground Line; by *Prop.* XIII.

That Lines *inclined* in the ground Plane, are to be drawn with the ſame Angle of Inclination, in the Table; by *Prop.* XV.

That the Parts of the Perſpective become *unequal*, and *ſhorten* more and more, the nearer they come towards the *Point of Sight*; by Prop. XIV.

Theſe Directions relate more ſpecially and immediately to the Practice; though thoſe which flow from the reſt of the *Propoſitions* and their *Corollaries*, are all of them

ſuch,

ſuch, as will be uſeful in ſome Caſe or other this Way. Thus (for Example) it may be of great Uſe to an Artiſt that deſires to be exact, to conſider what is ſhewn at *Prop.* XI, XII, XIII. with their Corollaries, about the *Augmentation or Diminution* of the Perſpective, upon the various Heights and Diſtances of the Eye: And to know in what Proportion of Magnitude, the Perſpectives of *parallel* and *perpendicular* Lines, are to be drawn upon the Table, to thoſe Lines themſelves.

And therefore, as theſe Uſes will be eaſily found out and made, by thoſe who ſhall take the right Courſe to join good Knowledge in *Geometry*, to this Part of *Opticks:* So I ſhall inſiſt no farther upon that Matter, but *come to propoſe ſome Problems*, ſuch as may ſerve to exerciſe the Rules before demonſtrated.

And for the more effectual attaining this End, we ſhall ſhew how they are to be done: 1. By *the more common and expeditious way of a Point of Sight, and a Point of Diſtance*; and how, 2. By the Help of the *Accidental Points.*

PROP.

PROP. XVII. PROB. I.
(Fig. 15. N°. 1.)

To find the Seat of a Point *in the Perſpective Table.*

By a Point of Sight *and* Diſtance.

Let the Eye be A, the Point of *Sight* B, the Point of *Diſtance* D, the Point whoſe Perſpective, or Seat in the Table, is required, F. The Line FD is perpendicular to the ground Line GR, wherein is taken ED = DF.

Then the *Radial* DB drawn from the Point of *Incidence* D, cuts the Line of Diſtance CE in *f*, which is the Seat of the given Point F, in the Perſpective Table.

By the *Accidental* Points. (Fig. 15. N°. 2, and 3.)

The *former* of theſe Figures will ſhew the Reaſon and Demonſtration of this Way of practiſing, by the Accidental Points, the Table being repreſented *ſideways*; and the *latter*, the more ready and expeditious Way of Practice it ſelf, the View being here *foreright*. And in both, the ſeveral correſpondent Points are mark'd exactly with the ſame Letters, that the Reader might the better underſtand the Agreement betwixt them.

Let the Eye be at A, its Height AB, the Plane of the Table CDEF, the ground Line EF.

The Difference between these Figures, is, that whereas the Object IKL at N°. 2. is represented very distinctly as lying in the ground Plane, and is not at all confounded, with its Image in the Table OPQ; at N°. 3. the Object *ikl* seems to be confounded with its Perspective *opq*, although they are not to be conceived, as both lying in the *same Plane*, but *ikl* out *behind* in the ground Plane, and *opq* in the Plane of the Table, erected perpendicularly upon the Line *ef*; the seeming Coincidence of the Planes, arising from the present Position and View.

Let I be a Point given in the ground Plane, (Fig. 15. N°. 2, 3.) whose Seat in the Table is to be determin'd.

From the given Point produce any two Lines, at Liberty, to cut the ground Line, as IN, IM.

From B the Foot of the Eye's Perpendicular, draw BF, BH, || to IM, IN, respectively. At the Points F, H, (in the ground Line) erect the Perpendiculars FD, HG, each equal to AB.

Join these Points D, G, with the Points M, N, respectively, and where the Lines DM, GN, intersect, as in O, will be the

Seat

Seat of the Point I in the Table. (This being demonſtrated at *Pro.* IX, and its *Corollaries*, I ſhall not need to offer any thing of the Reaſon of it here.)

PROP. XVIII. PROB. II. (Fig. 16.)

To find the Scenographick *Contraction of a* Right Line, *drawn in any Poſition* to the ground Line.

By a Point of Sight *and* Diſtance.

The Points A, B, C, as before. The Line given IH; and from the Points I, H, the Perpendiculars EI, H G, to which D E and FG, in the Ground Line GD, are reſpectively equal. From the Points of *Incidence* EG are drawn the *Radials* EC, GC, which are interſected by the Lines of *Diſtance* BD, BF, in the Points *i*, *h*, which Points are the Perſpectives of I, H, and therefore being joyned with a ſtrait Line, give *ih* for the Perſpective of the Line IH, which was ſought.

Were the given Line *parallel* to the ground Line, as *ex. gr.* IK; its Perſpective is determin'd with leſs Trouble ſtill. For having found the Seat of the Point I in *i*, we need only draw from thence a Line *parallel* to DG, which cuts the other *Radial* CG in *k*, and ſo makes *ik* for the Perſpective of IK. There

There is no Need to add any thing about the Perſpective of a Line *perpendicular* to the ground Line. The Figure it ſelf, ſufficiently ſhews that Matter; as *ex. gr.* in the Lines EI and GK, whoſe Perſpectives are Ei and GK.

To do the ſame by the *Accidental* Points.

1. If it be an *Infinite* Right Line, whoſe Scenographick Contraction we would have, as *ex. gr.* MK, infinitely produced towards K, and cutting the Table in M (Fig. 15. N°. 2, and 3.) we have nothing to do, but to draw BF || to MK, and having erected FD perpendicular to FE, and = AB, to join the Points D and M: So is DM the Perſpective of the infinite Line MK.

2. If a *Finite* Line, as IM, and *contiguous* to the Table; we need only draw any other Line at Liberty, as NI, thro' the Point I the Extremity thereof; and then determine (by the laſt *Prob.*) O, the Seat of I, in the Table: For then joining the Points M, O, that Line is the Perſpective Contraction of IM.

3. If a Finite Line, and not contiguous to the Table, as LI; this may be done two ſeveral Ways:

1. By

1. By *Two* Accidental Points.

This is done by drawing two Lines at Liberty thro' the Extremities of the given Line, as LK, KI, to cut the Ground Line in R and M, and producing LI it ſelf, to cut the ground Line in N; then having determin'd C, D, the proper *accidental* Points, to which the ſaid Lines run up in the Table, as alſo G, the Point to which the Line LI, infinitely produced, runs up; draw the Lines CR, DM, GN: For then CR and DM, do cut off from the Line GN, a Segment OP, which is the *Scenographick* Contraction of LI ſought.

2. By *One* Accidental Point only.

From the Extremities L, I, draw two Lines *parallel* to each other, till they cut the ground Line; and by what has been ſhew'd before, find the proper accidental Point, to which thoſe *Parallels* converge on the Table.

Then having (as before) produced LI to the ground Line in N, and found G the proper accidental Point, to which is runs up in the Table. If two Lines are drawn from the Point, to which the aforeſaid Parallels converge on the Table, to the Points where they cut the ground Line; theſe will

will cut off from the Line NG, the true Perſpective of LI.

PROB. III.

To repreſent any given Angle *in Perſpective.* (Fig. 15. N°. 1, 2, 3.)

This is ſo plain and eaſie an Operation, after what has been ſaid about *Points* and *Lines*; that there ought to be but few Words, made about it. *Ex. gr.* Suppoſe the Angle *kli* (Fig. 15. N°. 3.) were given. Having produced the containing Sides *lk*, *li*, to the ground Line in *r*, *n*, and carry'd Lines from *b* the Foot of the Eye's perpendicular, parallel to them reſpectively, *viz.* *be*, *bh*, and erected the Perpendiculars, *ec*, *hg* $=$ *ab*, and laſtly joined the Points *c* and *r*, *g* and *n* with right Lines: I ſay thoſe Lines *cr*, *gn*, form an Angle *cgp*, or *npr*, equal in *Scenographick Repreſentation*, to the Angle *kli*.

PROB. IV. (Fig. 18.)

To find the Perſpective of a Line, Perpendicular, to the ground Plane.

This Problem is abſolutely neceſſary, in order to the ſetting any ſort of *Solid* in Perſpective;

Perſpective; and therefore ought to be diligently explain'd.

Let the Perpendicular given be NM, its *Seat* in the ground Plane I, its Diſtance from the Table IB, the Height OB=NM, ſet off in the Table from the Point of Incidence B. The Lines OA, BA, *Radials* carried from the Points O, B. In the Radial BA, the Point C, is the perſpective of I, the Point of *Seat*. From C is drawn CD, parallel to OB, terminated in D by the Radial OA. The Line CD is the Perſpective of NM.

Or thus: From any Point in the ground Line, as E, ſet off the Perpendicular EF =BO=NM, and having determin'd the Point C (as before) draw CG parallel to the ground Line, to cut EA in G, then will GH perpendicular to the Ground Line, terminated by the Line FA, be the Perſpective ſought. Or (which is the ſame thing) GH and CD will be equal to one another.

For AB : AC : : OB : DC, Similar $\triangle^{ls}$ AOB, DOC, and FE : HG : : AE : AG, Similar $\triangle^{ls}$, AFE, AHG, and AE : AG : : AB : AC, Similar $\triangle^{ls}$ ABE, ACG,
$\therefore$ FE : HG : : AB : AC,
$\therefore$ FE : HG : : OB : DC,
But FE=OB, $\therefore$ HG=DC.

And it may be much more convenient ſometimes, thus to find the Perſpective of an upright Line, *apart from the reſt of the Perſpective*, and afterwards transfer it to its own proper Place in the Perſpective, then to ſet it off there at firſt.

For when the Caſe is ſuch, that a great many Perpendiculars are to be carry'd up from the Perſpectives of their ſeveral Points of Seat in the Figure; by the Multiplicity of Lines, the whole will be apt to be rendred confuſed.

If it were required to determine the Perſpective, of a Line, any way *inclin'd* to the ground Plane; it's readily done thus.

Let fall a Perpendicular from the Top of the *inclin'd* Line, to the ground Plane. Find the Perſpective of that Perpendicular; as alſo the Perſpective Seat of the Foot of the inclin'd Line.

Draw a Line from the Top of the aforeſaid Perſpective Height, to the Seat of the Foot of the given inclin'd Line, which will be the Perſpective ſought.

PROP.

PROP. XXI. PROB. V. (Fig. 19.)

To find the Perspective of a Triangle, in any Position to the Table.

By a Point of Sight and Distance.

Let KLM be the Triangle propos'd. KE, LH, IM, are Perpendiculars from the Angles, to the ground Line; and from the Points E, H, I, are drawn the *Radials* EB, HB and IB. The Perpendicular's KE, *&c.* laid off in the Ground Line, give the Points D, F, G, the Lines drawn to which from C, intersecting the Radials in *k*, *l*, *m*, determine the Perspective of the Triangle KLM.

By the Accidental *Points.*

Let the Triangle be KIL (Fig. 15. N°. 2, 3.) produce the Sides, to cut the ground Line in R, N, M, and then from B, drawing Parallels to them, in E, H, F, erect the Perpendiculars EC, HG, F D, and join the Points C, G, D, with R, N, M, respectively: So have we by the Intersections of these Lines, the Triangle O PQ in the Table, for the Perspective of IKL.

And it may be much more convenient ſometimes, thus to find the Perſpective of an upright Line, *apart from the reſt of the Perſpective*, and afterwards transfer it to its own proper Place in the Perſpective, then to ſet it off there at firſt.

For when the Caſe is ſuch, that a great many Perpendiculars are to be carry'd up from the Perſpectives of their ſeveral Points of Seat in the Figure; by the Multiplicity of Lines, the whole will be apt to be rendred confuſed.

If it were required to determine the Perſpective, of a Line, any way *inclin'd* to the ground Plane; it's readily done thus.

Let fall a Perpendicular from the Top of the *inclin'd* Line, to the ground Plane. Find the Perſpective of that Perpendicular; as alſo the Perſpective Seat of the Foot of the inclin'd Line.

Draw a Line from the Top of the aforeſaid Perſpective Height, to the Seat of the Foot of the given inclin'd Line, which will be the Perſpective ſought.

PROP.

PROP. XXI. PROB. V. (Fig. 19.)

To find the Perspective of a Triangle, in any Position to the Table.

By a Point of Sight and Distance.

Let KLM be the Triangle propos'd. KE, LH, IM, are Perpendiculars from the Angles, to the ground Line; and from the Points E, H, I, are drawn the *Radials* EB, HB and IB. The Perpendicular's KE, *&c.* laid off in the Ground Line, give the Points D, F, G, the Lines drawn to which from C, intersecting the Radials in *k*, *l*, *m*, determine the Perspective of the Triangle KLM.

By the Accidental *Points.*

Let the Triangle be KIL (Fig. 15. N°. 2, 3.) produce the Sides, to cut the ground Line in R, N, M, and then from B, drawing Parallels to them, in E, H, F, erect the Perpendiculars EC, HG, F D, and join the Points C, G, D, with R, N, M, respectively: So have we by the Intersections of these Lines, the Triangle O PQ in the Table, for the Perspective of IKL.

After this Inſtance, I ſhall not need to illuſtrate the Method of drawing Pieces of Perſpective, by the Help of the *Accidental* Points, in any other Figures whatſoever; unleſs perhaps where 'tis nearer and more expeditious, to work that way than the other.

PROP. XXII. PROB. VI. (Fig. 20.)

To repreſent in Perſpective, a Square, divided into ſeveral little Squares; one Side being parallel *to the ground Line.*

Let the Square be AKGD. Let AY, ZH, In, qK, Perpendiculars to the ground Line, in which the Diſtances being laid off, mark out the Points Q, R, S, T, V, W, X. Let the Lines YO, ZO, nO, qO, be *Radials.* The Lines PQ, PR, *&c.* carry'd from the Point of *Diſtance* C, interſect the Radial Oq, in *g*, *m*, *l*, *k*, from whence drawing *dg*, *me*, *lb*, *ka*, parallel to QR, we have the deſir'd Diviſions in Perſpective.

PROP. XXIII. PROB. VII. (Fig. 21.)

To do the ſame when one Angle *of the Square is turned to the Table.*

This is moſt conveniently done, by the Help of the *Accidental* Points.

The

The Lines of *Incidence* being drawn, and the Diſtances laid off in the Ground Line (as uſual) let nSWZ be the perſpective outlines of the Square MFNI. The Accidental Points are E and C, the Lines AE and AC, being parallel to MF and FI, by *Cor.* X. *Prop.* IX. Let the Points W, X, Y, Z, be the Perſpectives of I, H, G, F. Then laying a Ruler thro' them and the Point E, mark out the Points *s*, *q*, *r*, *n*, which joyned with W, X, Y, Z, will divide the Sides nS, ZW. So likewiſe, the Points *n*, *o*, *p*, Z, being found, a Ruler laid over them and the Point C, will divide the other two Sides of the Perſpective, *nz*, SW. And the Interſections of theſe croſs Lines, will determine the Perſpective of the little Squares, in the *Original*.

PROP. XXIV. PROB. VIII.

To ſet any Rectilineal Plane Figure, whatſoever, whether Regular, or Irregular, in Perſpective.

Find by the Rules afore-given, the Seats of the ſeveral *Angular* Points of the Poligon given, in the Perſpective Table. Theſe Points joyn'd with Right Lines, will give the Perſpective of the Figure propos'd.

PROP. XXV. PROB. IX. (Fig. 22.)

To ſet any upright *Priſm*, *or Pyramid in Perſpective.*

For a Priſm.

Let the Baſe be ABCDE, whoſe Side AB is parallel to the Ground Line, and the Height NO. By the foregoing Rules, find the Perſpective of the Baſe, which let be FGHIK, having drawn a Line from O, any Point in the ground Line, to L the Point of Sight; erect the Perpendicular NO equal to the Height of the Priſm, and join NL. At the Points F,G,H,I,K, a Ruler being laid parallel to the ground Line OZ. Interſects the Line LO, in the Points *g*, *h*, *i*, from whence drawing gR, hQ, iP parallel to NO; theſe ſhall be (by *Prop*. IV.) the perſpective Heights of the Priſm at thoſe ſeveral Points. Where note, that there are but three different Heights in all, for thoſe that are to be raiſed upon K and I, will be equal one to another, ſo likewiſe with thoſe at F and H, tho' leſs than the former. *Laſtly*, that at G, will be the leaſt of all.

The Reaſon of this is, becauſe the Side the Pentagon AB is ſuppoſed to be parallel to the ground Line. So that now laying down the perſpective Plane or Baſe FGHIK, in

in a Place apart by it ſelf; upon KI erect the Perpendiculars KP, IT, equal to the Perpendicular iP, and on the Points F, H, erect FQ, HS equal to hQ; and Laſtly, from G erect GR=gR, and join the Points PQRST, ſo you have the Perſpective of the whole Solid.

For a *Pyramid.*

Let the Baſe, as before, be ABCDE, the Height NO, and W the Center of the Baſe.

Having drawn the *Perſpective Plan* FGHIK, and therein determin'd *w* the Seat of W; we have nothing to do, but from thence to carry a || to the Ground Line, and at its Interſection with the Radial LO, to take off (as before) the proper Perſpective *Altitude*, which is afterwards to be erected upon the Point *w* in the Plan. For Lines drawn from the Extremity hereof, to Angles F, G, H, I, K, will compleat the Perſpective of the Pyramid.

PROP. XXVI. PROB. X.

To ſet any Sort of Oblique *Priſm or Pyramid, in Perſpective.*

For *Pyramids*; we ſhall need only one Perpendicular let fall from the *Vertex*, to the Ground Plane.

Having therefore drawn the perſpective Plan, and determin'd whereabout in the Table, the Seat of that Point in the Ground Plane, on which the Perpendicular from the *Vertex* falls, will be; as alſo having determin'd (by the Rules above given) the juſt Meaſure, of the Perſpective, of the *ſaid Perpendicular:* Laſtly, having ſet off this Perſpective Altitude, from its proper Seat in the *Plan:* There is no more to be done, but to draw Lines, from the Extremity thereof, to the ſeveral Angles of the aforeſaid Perſpective Baſe or Plan.

In *Priſms*, the Matter is a little more troubleſome, becauſe of the *many* Perpendiculars required to be let fall on the ground Plane, from the *upper* Angles of the Body. (As *eg. gr.* in the oblique Quadrangular Priſm CDFEGHIK (Fig. 22.) from whoſe upper Angles, are let fall the Perpendiculars CL, FN, DM, EO; and whoſe Side GH, and conſequently IK, for facilitating the Work, I ſuppoſe to be parallel to the Ground Line OQ.)

However, thoſe Perpendiculars being let fall, and the *Meaſures of their Perſpectives, pm, qo,* duly determin'd, as alſo *their Points*

Points of Seat m, b, n, o, in the Table: Then if thoſe Perſpective Altitudes *qo, pm*, be each ſet off, in its *proper Meaſure*, from its proper *Point of Seat* in the Table, *viz. pm* from *b* and *m*, and *qo* from *n* and *o*; and Lines, *viz. cg, dh, fi, ek*, drawn from the Extremities of them, *viz. c, d, f, e*, to the *correſpondent* Angles of the Perſpective Plan, or Baſe, *viz. g, h, i, k*; and *Laſtly*, if thoſe Extremities themſelves be *aptly joined* with Right Lines, *viz. cd, de, ef, fc*; the Perſpective of the oblique Priſm propoſed, will be compleated, *viz. ikghfecd.*

SCHOL.

There is in theſe Caſes, Choice to be made of ſome ſuch Poſition of a Body to the Table, *that the Work may be the eaſieſt and ſhorteſt poſſible.*

Thus for Example, the forementioned Priſm CDEFGHIK, (at Fig. 22.) was placed with its Side GH parallel to the Ground Line OQ; and conſequently the two entire Surfaces of it, GHCD, IKFE, parallel to the Plane of the Table. By this Means the two Perpendiculars CL, DM, being at equal Diſtances from the Table, are repreſented in Perſpective by one and the ſame Line *pm*. And ſo the other

other two FN, EO, by the Line *qo*. So that we have but these two Perspective Heights, to determine in this Case. Whereas, had one of the *Angles* as G, been turn'd towards the Table, we might have had three or four several Perspective Altitudes, to have determin'd. If the *Diagonal* IH or NM, had been parallel to QO; then there had been *Three*, of which, that for CL would have been biggest; those for FN and DM, less than the former, tho' equal to one another, and that for OE least of all, as being the farthest from the Table.

But if NM were not parallel to the Table, it is plain, there must have been *Four* several Perspective Heights found; since the four Perpendiculars above-mention'd, would in that Case have stood, at four several unequal Distances from the Table.

The like is to be observed, in other Figures.

PROP. XXVII. PROB. XI.

To set any Solid, contained under Plane Surfaces (whether Regular or Irregular) *in Perspective.*

The Operation for the *oblique Prism*, (in the last Problem) will be a sufficient Direction for this, without a particular Figure

gure. *Ex. gr.* Let the Body propoſed be an *Icoſaedron*, which we will ſuppoſe ſet on its Baſe, which is one of the *Twenty Equilateral* Triangles, under which it is contain'd. This Body having twelve ſolid Angles; when it is ſet on one of its containing Triangles as a Baſe; there are nine of the ſaid Angles, remaining above the *ground Plane*, from each of which, Perpendiculars are to be let fall. And here now we ſhall find, the Uſe of what was hinted at the *Scholium* of the laſt Problem; about the Choice of ſuch a Poſition, that the Work may be the ſhorteſt poſſible. For if the Equilateral Triangle, which is the Baſe, be turned with one of its Sides ||l to the Table; we ſhall have the Perſpectives of but *ſix* Perpendiculars to determine. And the ſame alſo, if one Angle of the Baſe be directed to the Table in ſuch ſort; that a Perpendicular let fall from thence to the oppoſite Baſe of the Equilateral Triangle, would, if produc'd, cut the ground Line at right Angles. For this is the ſame Caſe as the former. But if it be ſet in any other Poſition, we ſhall have *nine* ſeveral Perpendiculars to ſet in Perſpective.

Having therefore let fall Perpendiculars from the elevated Angles to the ground Plane, and ſet the Baſe (whereon the Body ſtands) in Perſpective; and *Laſtly* determin'd

termin'd the *proper Heights* of thoſe ſeveral Perpendiculars, upon the Perſpective Table, and ſet them off from their proper *Points of Seat* therein: If then the Points are aptly joyn'd (as the Inſpection of ſuch a Solid will beſt direct) the *Perſpective out-Lines* of the Body will be compleated.

And thus may any Body whatſoever, contain'd under Rectilineal Surfaces, be expeditiouſly ſet in *Perſpective.*

PROP. XXVIII. PROB. XII.

To ſet all Sorts of Cones *and* Cylinders, *in Perſpective.*

The Rule will proceed here in like manner as at *Prob.* X, for *Pyramids* and *Priſms*; abating only the Difference ariſing from the *Baſes*, which here are *Curvilineal* Figures, (*viz. Circles*) and there, *Rectilineal* ones.

At *Cor.* I. *Prop.* V. I have ſhewn how to determine, when a Circle, ſhall come an exact Circle, into the Perſpective Table: That is, when the Perſpective of a Circle ſhall be a Circle. And it muſt always be either a *Circle* or an *Ellipſis*, when the Table ſtands as we now ſuppoſe it, *viz.* Perpendicular to the Ground Plane.

For

For that the Perſpective of a Circular Line, may upon other Suppoſitions, be any other of the Conical Sections, as well as an Ellipſis ; we have ſhewn already at *Prop.* VI.

So that therefore if *ſuch Diſtance and Height of the Eye*, be made Choice of, that the Baſe of the given Cone or Cylinder, be a *Circle* in the Perſpective Table ; if the *Seat* of the Center, and the perſpective Magnitude of the Radius, be likewiſe determin'd, (by *Prob.* I. and II.) the Baſe is deſcrib'd with little Trouble.

But if any other Poſition be choſen, ſo that the circular Baſe, of the Cone or Cylinder, comes into an *Ellipſis* upon the Table ; it may be deſcribed ſufficiently well for common Practice ; by dividing the Circumference of the Circle into a good Number of Parts, and having found the Perſpectives of the ſeveral Points of the Diviſion, *to carry a Crooked Line thro' them*, with a ſteddy Hand. Or to go more *Geometrically* to work ; the Ellipſis may be deſcrib'd, by finding the Longer and Shorter Axes of it upon the *Table* (as ſhall be ſhewn by and by) or by many other Ways beſides.

SCHOL.

SCHOL. I.

One thing is to be minded here with Regard to *Cylinders* (for there is no farther Difficulty at all in *Cones)* and that is, that tho' ſuch a Poſition be made Choice of, that the *Lower* Baſe, *ex. gr.* ſhould be a Circle in the Table, yet the *Upper* Baſe cannot at the ſame time, be ſo too, but will be an Ellipſis: Or *vice verſâ*, if the Upper be a Circle, the *Lower* will be an Ellipſis.

The Reaſon of which is moſt evident, from that aforeſaid *Cor.* I. *Prop.* VI.

For ſince the *Diſtance* from the Table, being *given*, there is *a particular Altitude* of the Eye required, in order to make the Perſpective of a Circle, to be a Circle; and ſince in an *Upright Cylinder*, the Upper and Lower Baſes are both equally diſtant from the Table, but the Eye has not an equal Elevation over them both: 'Tis plain, that if the *Height* of the Eye over the *Lower* Baſe in the ground Plane, be ſo proportion'd to its Diſtance from the Table, that the Perſpective thereof ſhall be a Circle; the *Leſs* Height of the Eye over the *Upper* Baſe, cannot be proportion'd to the ſame Diſtance from the Table, ſo as to produce the ſame Effect.

So

So that in an upright Cylinder, the Perſpective Appearances, of the upper and lower Baſis, can never be of the ſame Kind, but if one be a *Circle*, the other will be an *Ellipſis*; that is, ſuppoſing the Cylinder it ſelf, and the *Eye*, to retain the ſame Poſition, and Diſtance from the Table.

SCHOL. II.

To find the longer and ſhorter Axes, of this Ellipſis, upon the Table.

Imagine two Diameters in the Circular Baſe of the Cylinder, cutting each other at Right Angles, ſo that one of them be parallel to the Table, and the other conſequently perpendicular thereto.

The Perſpectives of theſe two Diameters, found by *Prop.* XI, and XII. will be the Axes of the Ellipſis to be deſcribed upon the Table.

Now the Length of the Diameter being given, the Scenographick Contractions thereof, in theſe two Poſitions, are eaſily found; by knowing the Diſtances it lies at from the Table, *viz.* the ſingle Diſtance of the Diameter which is *parallel*, and the Diſtances of the two Extremities, of that, which is *perpendicular*, to the Table.

Theſe

These, I say, being given, the Perspectives are found, by the Rules aforemention'd; and consequently, the Axes determin'd.

PROP. XXIX. PROB. XIII.

(Fig. 23, 24, 25.)

To set a Row of Bodies in Perspective.

We will take a Series of *Parallelipipids*, and suppose them rang'd in such Order, on one side the Eye, that their Sides which are perpendicular to the Table, may lie all in a right Line. And we will imagine one of them to be *contiguous to the Table*; which will in some Measure shorten the Work.

Let (Fig. 23.) the Point of *Sight* be at A, the Point of Distance B, Z the *Square* Base, and WQ one of the Including Rectangles of the Parallelipipid propos'd.

Make CD = WX, DH = XQ, as also HW = DH. Draw the Lines HA, DA, CA, from the Points H, D, C, to the Point of Sight A.

Joyn the Points BW, with a right Line, intersecting the Radial AH in G, from whence a parallel to the Ground Line DWZ, cuts the Radial AD in F, and determines the Trapezium HDGF, for the Perspective Base of the first Parallelipipid.

Lastly,

Lastly, from F raise a Perpendicular to DZ, (or which is all one, a Parallel to DC, which we suppose perpendicular to DZ) which intersects the Radial AC in E; and thus we have the diminish'd Altitude EF, and consequently DCEF for the Perspective Representation, of the Side-Rectangle or Surface of that first Parallelipipid.

And from this Base and Side-Superficies, all the rest that finishes the Perspective of the said Body, is determin'd.

And thus are the Bases and Sides found for the rest, *viz.* OMNI, LMIK, for the Second,; VSQT, RSPQ, for the Third, and so on.

Note, If the first Parallelipipid had not been suppos'd contiguous to the Table, the Side DH, could not have lien in the Ground Line, but would have been at some Distance from it, and so would have been *diminished*, and not appear'd in its full Bigness, as now it does.

And the Reason, why we made HW =HD, is because the Base Z being suppos'd a Square, that side thereof which lies opposite to DH, cannot be distant from the Table any more or less, than the Length of DH it self; for (as I said) this first Parallelipipid is *contiguous* to the Table.

Nor is there any other Difference in the Work, when one Body is *placed contigu-*

ous to the Table, and when they are all *at a Distance* from it, than only this, that in the former Case 'tis shorter, by as much, as finding the Perspective of one Line amounts to.

The Perspective *Plans* and *Elevations* being found, as at Fig. 23. the Perspectives of the *whole Solids* are set together very easily, as at Fig. 24. *Ex. gr.* The Plan GFHD, Fig. 23. being transferred to *gfhd*, Fig. 24. upon the Points *h*, *d*, erect the Perpendiculars *dc*, *ah*, = DC, and at *g*, *f*, the Perpendiculars *gb*, *ef*, = EF, and joyning the Points at top and bottom, with right Lines, (as in the Fig.) the out Lines of the Parallelipipid are compleated. In like Manner for the Second and Third, transfer the Plans OMNI, VSTQ, into *o m n i*, *v s t q*, and erect the Perpendiculars KI, LM, in the Second, and PQ, RS, in the Third, each in its proper Place; and so fill up the out Lines for them, and the rest, if there were more.

Lastly, The Parallelipipids compleatly *finish'd and shaded*, appear as at C, D, E, Fig. 25.

PROP.

PROP. XXX. PROB. XII.
(Fig. 26.)

To represent a Pedestal, *in Perspective.*

This is done very easily, by Help of the Directions given at the last Problem.

Let the *Geometrick Plan* or *Base* be F, the Geometrick Elevation or *Profile* C, the Point of *Sight* at A, and of *Distance* B; he Lines CK, and HK perpendicular to ach other.

Having put the Base F into Perspective, s at E, and drawn the occult Lines *b*, *b*, &c. from the several Angles of the Ele-ation C, perpendicular to HK, as also e occult Lines *c*, *c*, &c. parallel to CK: uppose the *Perspective* Elevation D, to be ompleated.

This being set in two opposite Sides f the perspective Plan, as was done for e *Parallelipipids* (at Fig. 24.) will com-leat the Perspective outlines of the Pe-estal.

And the whole adorn'd with its *proper hades*, will appear as at G.

PROP. XXXII. PROB. XIII.
(Fig. 27.)

To delineate the Perspective Representation of the Roof, Pavement *and* Side-walls, *of any long Room or Entry, whose Dimensions are given.*

Suppose BC the Height, CD the Breadth, CH the Length, of the *Place* propos'd, IE the *Eye's* Height, FE its Distance; al taken off in their proper Measures, from th same *Scale* of equal Parts. The *Table* i imagin'd to stand perpendicular upon CD (which is therefore our *Ground Line*) an the Spectatours Position such, that a Per pendicular from his Foot to CD, fal thereon at the Point F; which Perpedi cular is EF.

Having drawn the *Rectangle* BCAD one of whose Sides is the Height, and th other the Breadth (and is the *Geometric* Section of the Room by a *vertical* Plan Breadth-ways) produce the Line EF, ti it cuts BA in K. In the Line FK, set o the Eyes Height IE from F to N, an draw the Lines NC, ND.

By *Corol.* I. *Prop.* XI. find the Persp ctive Contraction, of the *Length* of t Place, *viz.* GH; saying,

As the Sum of the *Length*, and the *Eyes Distance* from the Table,
Is to the *Height* of the Eye;
So is the *Length* it self,
To its Perspective *Contraction*, or Foreshortning.

That is, GH+FE : IE : : GH : to a Fourth, or the *Foreshortning* sought.

This being taken off, from the *Scale* used before, for the Geometrick Delineation, is to be laid in the Line FN, from F *ex. gr.* to L. Thro' the Point L, draw PO parallel CD, intersecting the Lines NC, ND, in the Points R, S.

Again, By *Corol.* II. *Prop.* XIII. find the Perspective Contraction, of the Rooms Height BC; saying,

As the Sum of the *Length*, and the Eyes Distance,
Is to the *Eyes Distance*,
So is the *Height* it self (of the Place)
To its Perspective *Contraction*.

That is, GH+FE : FE : : BC : to a Fourth, which is the *Contraction* sought.

Let this be taken off from the same Scale, and laid in the Line FK, from the Point L (determin'd before) to W *ex. gr.*

or which is the ſame thing, ſet it off in the Lines RQ, ST, which are parallel BC, from the Points R, or S, to Y, or X. Which done, compleat the Rectangle RSYX, and draw the Lines BY, AX. Or elſe, having ſet off LW, thro' W draw NM parallel to BA, which cuts the Lines NB, NA, in the Points Y, X, and ſo does the very ſame thing. And thus all the outlines, of the intended Peice of Perſpective, are drawn.

For the Trapezium CRSD, is the Repreſentation of the *Floor*, BYXA of the *Roof*, BYRC and AXSD of the *Side Walls*.

PROP. XXXII. PROB. XIV. (Fig. 28.)

To repreſent an Arch *in Perſpective.*

This Work is ſo like that of the foregoing Problem, that there need not be much ſaid of it.

The Eye's *Height* is ſet off from N to L. The Line MN is the Perſpective Contraction of the *Length* (or Depth) and GM, of the *Height*; anſwering to LF, and WL, in the *laſt Figure*, and obtained by the ſame Proportions. The Figure HGKPV is here in this Caſe, what YXRS was in that; the Circular Arch, being carry'd thro'

thro' the three Points H, G, K, which are determin'd by the above-mention'd Rules, which give the foreshortning of the Length and Heights.

The Lines BH, *nm*, EK, *rq*, OW, TV, YZ, PR, are directed to the *Point of Sight* L; as in the former Case, CR, DS, BY, AX, were carry'd towards N. In a Word; TVPR is the Perspective Representation of the *Ground Area*, BHTV and FKPR of the *Sides*, and HBAEK, of the *Concave Superficies* of the Arch.

And by the Help of these two Examples, may any other Delineations of the like Nature be perform'd.

PROP. XXXIII. PROB. XV. (Fig. 29.)

To perform the Practice of Perspective, without Regard to Point of Distance, or any Accidental Point whatsoever.

Let the Table be ABDI, the Point of *Sight* C, the Eyes Height CE, any visible Point in the Ground Plane, P, whose Incidence on the *Ground* Line, is at H, and its Distance PH.

CONSTRUCTION.

Draw the Line DO in any Angle at Liberty to DI, Make DN (*ex. gr.* or DL,

if it happened to be less) equal to the Eye's *Distance* from the Table, NO (or LM) = PH, the Distance of the given Point from the Ground Line. Draw OI (or MI) and NK (or LK) parallel thereto. Thro' K draw KS parallel to BI, and lastly CH intersecting KS in Q.

I say that Q is the proper Place or Seat of the Point P, in the Perspective Table.

DEMONSTRATION.

Call the Perspective of the Line PH, π.
By *Prop*. XI. $PH : \pi :: DN + PH : CH$,

$\therefore DN + PH : PH :: CH : \pi$,
$\therefore DN : PH :: CH - \pi : \pi$,
But $NO = PH$ (*Construct.*)
$\therefore DN : NO :: CH - \pi : \pi$.

Again, $DN : NO :: DK : KI :: CQ : QH$ (Similar $\triangle^{ls}$.)

$\therefore CQ : QH :: CH - \pi : \pi$,
$\therefore QH : CH :: \pi : CH - \pi + \pi = CH$,
$\therefore QH = \pi$.

Therefore Q is the Seat of the given Point P, in the Perspective Table. *Q. E. D.*

SCHOL.

SCHOL.

'Tis sufficient, to have shewn the way of tracing out by this Method, the Seat of any Point in the Perspective Table; since from hence any Figure whatsoever may be easily laid down.

But the Ways for doing these things are endless; and therefore I shall leave it to every one, to invent or follow what Method he pleases.

Having now dispatched what Problems are necessary, to render any Studious Person sufficiently well acquainted with the Practice of the *Direct* Method of Perspective, upon *Upright or Vertical* Tables.

I shall add one or two Propositions, tending to the farther Illustration, and Improvement, of this curious Subject; and then come to shew how we are to proceed upon *Horizontal* and *Inclin'd* Tables.

After that, in a few Instances, I shall exemplifie the *Inverse* Method of Perspective; that is, how to go back from the *Perspective*, to the *Original*, or Object, whose Perspective it is. And the foregoing Rules being well understood, there will be no Need, to say much upon that Matter.

PROP. XXXIV. THEOR.
(Fig. 7.)

Every Deformation, is a regular Piece of Perſpective, *upon the very ſame Plane; to the Eye, plac'd at another Height and Diſtance.*

I have already ſaid ſomething in general at *Prop.* VII, and its *Coroll.* concerning the Nature of *Optical Deformation*, and its Diſtinction from what we commonly call *Perſpective.* It is ſhewn there, as alſo at *Schol. Prop.* VI. that this is no other than an *inverted* Sort of Perſpective; and that upon the Account of the *different Order*, in which the Object and the Table lie, with reſpect to the Eye.

But I ſhall now demonſtrate other Reaſons for its being ſo, and ſhew how theſe Practices, do all fall within the Rules of the ordinary Perſpective.

CONSTRUCTION.

Let the Eye be at K, its Height KV, the *Ground Plane* (which ſerves as a *Table* in this Caſe) VSTDE; on which is perpendicularly erected the Plane Figure ABPN, which is projected by the Eye at K, into PNDE. We

We will suppose the Figure ABPN to be a Rectangle, and consequently, its Deformation E N DE, is a Trapezium; whose Side PN is parallel DE.

Upon DE erect a Plane, as MQFH, perpendicular to the Ground Plane, which produce out both Ways at Liberty. Let fall a Perpendicular thereto, from the Eye at K; which cuts it in the Point G.

DEMONSRATION.

The Lines PD, NE, concur in V, the Foot of the Eye's Perpendicular (by *Construction*, at *Prop.* VII.) And since the Plane MQFH is (by the Hypothesis) perpendicular to the Plane STDE; therefore if the Former be made a *Ground Plane*, and the Latter a *Perspective* Table; its evident that the Parallels MF, QH infinitely produced, will run up to some Point of *Sight*, in the aforesaid Plane STDE. Now if KG be made the Height of the Eye, and KV its Distance from the Table; then the Point of *Sight* is V, and the Lines DV, EV, the Perspectives of the Parallels DF, EH, infinitely produced; and therefore PD, NE, are the Perspectives of some Finite Portions of those Parallels. Therefore the *Eye* being at K, the Ground Plane MQFH, the *Table* VSTDE, the *Eye's Height* KG, its

its *Distance* KV; the Trapezium PDNE, is the true *Perspective*, of some Portion of the Rectangle DEFH produced. But the same Trapezium was the Deformation of the Rectangle ABPN, to the same Eye at K, its Height being KV, its Distance VO, and the Table STDE. Therefore, *&c.* *Q. E. D.*

SCHOL. I.

We may easily determine, what Part of the Infinite Rectangle, FDEH, the Deformation PNDE, is the common Perspective of, upon the Table VSTDE.

For drawing AI or BI, parallel to VD or VE, and VO perpendicular to PN, and produced to cut DE in T; from Similar $\triangle^{ls}$, arises KI (=KV—IV=KV—BN) : IB (=VN) : : KV : VE.

Again, VN : PN : : VE : DE; and Lastly, VE—NV=NE.

Now then, if we take DE for the *ground Line*, and erect the Infinite Plane VSTDE perpendicularly thereon, as our *new Table*, and also at Right Angles there to the Infinite Plane MQFH, for our ground Plane; we have then the *Height*, and *Distance* of the Eye, as also the *Perspective* Contraction of some Portion of a *Direct* Line to find (by Prop. XI.) the Length of that Direct Line it self. That is, we have KG (=VT) and

KV,

KV, and NE, to find the *Length*, of which NE is the Scenographick Contraction.

And this being found, we may pronounce, *viz.* that a Rectangle one of whose Sides is DE, and the other the Line thus found, being put into Perspective by the commo Rules, for the *Height* and *Distance* of the Eye, GK, and KV; will be the true *Deformation* of the given Rectangle AN, upon the same Plane, but with the Height and Distance of the Eye, KV, and VO.

SCHOL. II.

Since *Shadows* are nothing but the *Deformations*, or *Projections of the Out-lines*, of Bodies, upon certain Planes; and since we have demonstrated, how the Practice of Deformations is reducible to that of the *Common Perspective*: 'Tis plain, that the Practice of *Sciagraphy*, or of determining the Shadows of Bodies, is likewise reducible to the same; so that from the necessary Data (of the *Figure* of the Body, and) of the *Height* and *Distance* of the *Light*, we may settle the proper Height and Distance of the *Eye*, that the Shadow may be drawn upon a *Table*, as an ordinary Piece of Perspective.

LEM.

LEM. (Fig. 30.)

If there be any Number of Planes, cutting each other in the ſame Right Line; and another Plane be drawn Perpendicular to their common Section: Then, the common Sections of the former *Planes with this* laſt *Plane, ſhall be all at right Angles, to the common Section of thoſe ſaid Planes.*

The Planes MLGQ, MKFP, MIEO, whoſe Ground Lines, LQ, KP, IO, are ſuppoſed Parallel; cut each other in the Line MBN, and are all of them cut, by the Plane ACD, in the Lines BE, BF, BG, reſpectively. The Line MB, is ſuppoſed perpendicular to the Plane ACD, at the Point B. Theſe things ſuppoſed; I ſay that MB, the common Section of the aforeſaid Planes with one another, ſhall be at right Angles to the Lines BE, BF, BG, the common Sections of the ſame Planes, with the Plane ACD.

This is ſo manifeſt from *Eucl. Elem.* 11. that there is no need of inſiſting on the Proof of it.

COROL.

The Triangles MBR, NBR, MBT, NBT, *&c.* lying in the Planes MLBGQ, MKBFP, *&c.* are all of them Rectangular at B.

PROP.

PROP. XXXV. THEOR.
(Fig. 30.)

If an Object in the Ground Plane, appear to the Eye, in Direct *Vision, in any Points whatsoever of an* Upright *Perspective Table; then, if a Plane Speculum, were substituted instead of the Table, and the Eye placed at the same Distance, on the other Side thereof; it would receive the true Perspective of the Object, by this* reflex'd *Vision, as before by the* Direct.

DEMONSTRATION.

Let the Eye be at M, the Table ACDH, any Object as OPQ, in the Ground Plane; whose Perspective, or Image in *Direct* Vision, is STR.

The common Sections, of the Planes MLGQ, MKFP, MIEO, with the Plane of the Table, *viz.* GR, FT, ES, do all run up to the Point B; as has been demonstrated at *Prop.* IX.

And (by the foregoing *Lemma*) these Lines RB, TB, SB, are each of them perpendicular to MB, the common Section of all the Planes.

Let us suppose then in the next Place, that the Table ACDH were a *Plane Speculum*,

lum, and that the Visual Rayes PT, QR, OS, were reflected thereby, into the Lines Tq, Sr, Rp, at the Points T, R, S; at which Points we imagine Tc, Rb, Sa, to be erected perpendicularly, to the Plane of the Speculum; and consequently to lie in the Planes MKBFP, MLBGQ, MIBEO. And let the Line MB, which is by the Supposition perpendicular to the Plane of the Glass, be produced out in the other Side at Liberty, as BV. By the known Laws of *Catoptricks*, the Angle PTc = cTq, QRb=bRp, OSa=aSr. But because Rb *ex. gr.* is perpendicular to the Plane of the Speculum, therefore the Angles bRB and bRG, are right ones. And consequently, the Angle BRp=the Angle QRG. But QRG = MRB; therefore MRB= BRp.

Let Rp cut the Line MV, in N.

Then, since MBR=NBR (being right ones, by *Corol.* to *Lem.* foregoing) and MRB=NRB, and BR common to both Triangles; 'tis plain that MB=BN.

In like Manner, we will demonstrate; that the Angle BTM=BTN, and BSM =BSN.

And consequently, that the other *reflex'd* visual Rayes, Tq, Sr, do also meet in the same Point N.

And

And therefore were the Eye placed at N, it would ſee the Object OPQ, by the Means of the Glaſs; appearing at *opq*, on the other Side, juſt as OPQ it ſelf appears, on this Side.

That is; as the Eye at M, ſees the Object OPQ, in *Direct* Viſion, appearing on the Table, as SRT; ſo the Eye at N (equally removed) ſees the ſame Object, by the Help of the *Glaſs*, appearing at *opq*, juſt as far *behind* the Glaſs, as OPQ is *before* it, and in the very ſame Form too, *viz.* That of SRT, which is the ſame Perſpective. *Q. E. D.*

COROL.

Hence *Plane Looking-Glaſſes*, may be uſefully apply'd, to the Purpoſe of drawing Pieces of Perſpective.

PROP. XXXVI. THEOR.

(Fig. 31.)

Images, formed by Reflexion from Plane Glaſſes, are regular Pieces of Perſpective; in which the Height, and Diſtance of the Eye, as alſo the proper Point of Sight, are all *eaſily determinable.*

This Propoſition, differs much from that which went before, For what was ſhew'd

there, was this; that the Eye, by the Help of a Plane Glaſs, might have the very ſame true Perſpective of an Object, which it would have, for any given Height and Diſtance of the Eye, in *Direct* Viſion.

But what is to be proved here, is this; that a Light being plac'd before a Plane Glaſs, the Image of the ſame Glaſs, form-ed by the reflex'd Light; *ex. gr.* upon the Roof or *Cieling* of a Room, will be a regular Piece of Perſpective, whoſe Point of Sight is ſomewhere determinable, upon the aforeſaid Roof or Cieling. So that the Looking-Glaſs, is here, not only the *Inſtrument* to reflect, but alſo the *Object* it ſelf, whoſe Form is reflected.

For as in other Caſes, a Speculum receiving the Species of ſome ordinary Object, reverberates it, and makes that Object viſible to the Eye at a proper Diſtance and Poſition; ſo here, a Speculum receiving the Rayes of an *actual Light*, or Luminous Body; returns it own Form or Shape, upon a neighbouring Plane; which will be very different, according to the Poſitions of the *Glaſs*, the *Plane*, and the *Luminary* it ſelf.

DEMON.

DEMON.

Let ABCD be a plane Looking-Glaſs, *ex. gr.* of a Rectangular Form, the Light at E, which falling on the Glaſs in the Rayes EC, EA, EB, ED, is reflected up to the *Ceiling* TNOL; and figures there, the Speculum it rebounded from, in the Form of a Trapezium *abcd*, whoſe two Sides, *ex. gr. ab*, and *cd*, are parallel to each other.

Suppoſe the Plane of the Glaſs, if continued, to cut the *Ground-floor* in the Line ST, which is croſs'd at right Angles in I, with the Line EF, at one End of which, ſtands the Light E.

Take IF (*behind* the Glaſs) = IE, the Diſtance of the Light *before* it; and from the Point F, erect a Perpendicular to the *Floor*, *viz.* FG, which ſtrikes the *Ceiling* in G.

I ſay G is the proper *Point of Sight*, for the Perſpective *cabd*; or 'tis that in which the converging Sides of the Trapezium, *ca*, *db*, if produced, would meet.

By *Prop.* XXXIV. *Theor.* If the Eye be plac'd at F, and were ſuppoſed to project the Object ABCD upon the Ceiling TNOL (which we ſuppoſe parallel to the Horizon) the *Deformation* $\alpha\beta\kappa\delta$ will be a regular

Piece of Perspective, upon the very same Plane; in which the Point of Sight will be G, and the Eyes *Distance* from the Table, FG, and its *Altitude*, a Perpendicular from F to a *vertical* Plane passing thro' $\kappa\delta$.

And *vice versâ*, the Perspective $\alpha\beta\kappa\delta$ upon the Ceiling, appears to the Eye at F, as the Rectangle ABCD, upon the Vertical Plane ABST.

But by *Prop.* XXXV. *Theor.* the Speculum being ABCD, if instead of the *Light*, an Eye were placed at E; it would receive the same Appearance, of the Object, $\alpha\beta\kappa\delta$, by this *reflex'd* Vision, at E, which it had before in *direct* Vision, when plac'd on the other Side at F; the Distance IF being = IE.

That is, the Rayes $F\alpha$, $F\beta$, $F\kappa$, $F\delta$, would be reflected into EA, EB, EC, ED. And therefore, on the other Hand, if instead of the Eye, a Light be placed at E, the Incident Rayes EA, EB, *&c.* will be reflected by the Glass ABCD, into $A\alpha$, $B\beta$, $C\kappa$, $D\delta$, which if produced would all meet in F.

So that 'tis plain, the Projection $\alpha\beta\kappa\delta$, and the reflected Image *abcd*, perfectly coincide with one another. And therefore the said reflected Image *abcd*, is a regular Piece of Perspective, whose Point of Sight is G. *Q. E. D.*

COROL.

COROL.

Hence again, Plane Glasses may be applied to Perspective Uses; but after a manner very different, from what was suggested at *Corol.* of the foregoing *Prop.*

Of Horizontal Perspective.

Tho' we have hitherto been professedly considering only *Upright* Tables, and how to trace the Appearances of Objects on them; yet the *last Proposition* intimates so much of the Reason of the Practice on *Horizontal Tables* likewise, that we have not only a very easie and natural Transition from thence, to this Speculation, but shall also find it necessary to say less of that Matter than otherwise, upon the Score of what we have there demonstrated.

PROP. XXXVII. THEOR. (Fig. 32.)

'Tis the same thing to draw a Piece of Perspective, upon an Horizontal *Table; as upon a* vertical *Table, the Eye's Height and Distance being alternately charged.*

DEMONSTRATION.

Let the Eye be at A; GO, the Ground Plane, AG its Height above the ſame; BE an Horizontal Plane above the Eye, DH a Plane perpendicular to the two former, AB, the Eye's Diſtance from the *Horizontal* Plane, AN, its Diſtance from the *Vertical* Plane, D any viſible Point in the Plane DH.

From the Eye at A draw the viſual Ray AD, cutting the *Horizontal* Table in C.

'Tis plain that C, is the Perſpective of the Point D (lying in the vertical Ground Plane ED) in the *Horizontal* Table BE, to the Eye at A, whoſe Diſtance from the Table is AB, and from the vertical Ground Plane, is AN=BE.

And therefore ſince the Angle AND is a right one; if, while the Eye continues ſtill in the ſame Point at A, we ſuppoſe HD, which was before a *vertical* Ground Plane, now to become an *Horizontal* one; as alſo AN and BE, which before were Horizontal, now to be ſet perpendicular to the Horizon: It is evident, that by this Change of Poſition, all things are now reduc'd to the common Caſe of *Upright* Tables.

For

For DH is the Ground Plane, BE the Vertical or Upright Table, AN the Height of the Eye above the Ground Plane, AB its Diſtance from the Table, and B the Point of Sight thereon.

And in either Caſe, the Point C, the Perſpective of D, continues in exactly the ſame Place and Poſition in the Table. And for the ſame Reaſon, would the Perſpectives of any other Points in the Ground Plane HD, be the ſame when BE ſtands vertical, as when it lies parallel to the *Horizon*.

And therefore, 'tis the ſame thing to draw a piece of Perſpective upon, *&c.* *Q. E. D.*

COROL. I.

The Rule therefore for Practice, is this, *viz.* To draw upon the *Horizontal* Table BE, after the manner that we would do, if it were an *Upright* one; wherein B ſhould be the *Point of Sight*, AN the Eye's *Height*, and AB its *Diſtance* from the Table.

COROL. II.

The ſame Rules hold, whether Pieces of *Perſpective* of this Kind, are to be drawn on Planes above or below the Eye; as *ex. gr.* whether on the *Roof* or Ceiling of a

Church, ſo as to be view'd from the Floor, or on the *Pavement*, ſo as to be view'd from a Gallery.

COROL. III.

Were a *Pedeſtal* or *Column* (or a Rank of each) ſtanding perpendicular to the Horizon, to be repreſented in this ſort of Perſpective, *ex. gr.* upon the *Cieling*; it would be the ſame thing, as to place the ſame *Pedeſtal* or *Column*, parallel to the Horizon in the Ground Plane; and then draw the Perſpective of it, upon an *Upright* Table, *ex. gr.* a *Wall*.

For thus; if we ſuppoſe, the Line DE (for Example) to be a Pillar, perpendicular to the Horizon GO; it will be all one, to repreſent this in Perſpective upon the Cieling BE; as it would be, if DH being the Horizon or Ground Plane, and conſequently the Pillar DE lying flat thereon: we ſhould draw the Perſpective of it, upon the *Wall*, or vertical Table BE.

And it is to be obſerved; that in either Caſe, the *Circles* keep their proper Form in the Perſpective; as lying in a Poſition parallel to the Table, and conſequently (by *Prop.* V.) being Circles there likewiſe.

And the *Sides* of the Columns, are *Direct* Lines, or ſuch as are perpendicular to the Table,

Table, and therefore in the Table are carried up to the Point of *Sight* B.

And upon this Account *Horizontal* Perſpective is indeed much eaſier, than *Vertical*, or that which is perform'd upon an Upright Table; contrary to what the *Painters* generally imagine. For 'tis plain, that 'tis eaſier, *ex. gr.* to put a Column, that lies flat on the Ground Plane, into Perſpective, upon an Upright Table; then 'tis to draw the Perſpective of that ſame Column, ſtanding perpendicular to the Ground Plane, upon the ſame vertical Table. For the Difference lies here; That in the *former* Caſe, the *Circles* (as I ſaid) keep their Form in the Table; and the *Sides* likewiſe, are all carry'd up to the Point of *Sight*; whereas in the *latter*, the Sides are to be *ſhortned* upon the Table, and alſo the Circles cannot keep their Form; for the Reaſon of which, I refer to *Schol.* I. *Prop.* 5. But now; we have ſhewn, that 'tis the ſame thing to repreſent an *Upright* Column, in Perſpective, upon on *Horizontal* Table; that 'tis to repreſent that ſame Column, lying flat in the Ground Plane, upon an *Upright* Table. And therefore, I ſay the Practice of *Horizontal* Perſpective, is in many Reſpects much eaſier than that of *Vertical*.

SCHOL.

Church, ſo as to be view'd from the Floor, or on the *Pavement*, ſo as to be view'd from a Gallery.

COROL. III.

Were a *Pedeſtal* or *Column* (or a Rank of each) ſtanding perpendicular to the Horizon, to be repreſented in this ſort of Perſpective, *ex. gr.* upon the *Cieling*; it would be the ſame thing, as to place the ſame *Pedeſtal* or *Column*, parallel to the Horizon in the Ground Plane; and then draw the Perſpective of it, upon an *Upright* Table, *ex. gr.* a *Wall*.

For thus; if we ſuppoſe, the Line DE (for Example) to be a Pillar, perpendicular to the Horizon GO; it will be all one, to repreſent this in Perſpective upon the Cieling BE; as it would be, if DH being the Horizon or Ground Plane, and conſequently the Pillar DE lying flat thereon: we ſhould draw the Perſpective of it, upon the *Wall*, or vertical Table BE.

And it is to be obſerved; that in either Caſe, the *Circles* keep their proper Form in the Perſpective; as lying in a Poſition parallel to the Table, and conſequently (by *Prop.* V.) being Circles there likewiſe.

And the *Sides* of the Columns, are *Direct* Lines, or ſuch as are perpendicular to the Table,

Table, and therefore in the Table are carried up to the Point of *Sight* B.

And upon this Account *Horizontal* Perſpective is indeed much eaſier, than *Vertical*, or that which is perform'd upon an Upright Table; contrary to what the *Painters* generally imagine. For 'tis plain, that 'tis eaſier, *ex. gr.* to put a Column, that lies flat on the Ground Plane, into Perſpective, upon an Upright Table; then 'tis to draw the Perſpective of that ſame Column, ſtanding perpendicular to the Ground Plane, upon the ſame vertical Table. For the Difference lies here; That in the *former* Caſe, the *Circles* (as I ſaid) keep their Form in the Table; and the *Sides* likewiſe, are all carry'd up to the Point of *Sight*; whereas in the *latter*, the Sides are to be *ſhortned* upon the Table, and alſo the Circles cannot keep their Form; for the Reaſon of which, I refer to *Schol.* I. *Prop.* 5. But now; we have ſhewn, that 'tis the ſame thing to repreſent an *Upright* Column, in Perſpective, upon on *Horizontal* Table; that 'tis to repreſent that ſame Column, lying flat in the Ground Plane, upon an *Upright* Table. And therefore, I ſay the Practice of *Horizontal* Perſpective, is in many Reſpects much eaſier than that of *Vertical*.

SCHOL.

SCHOL.

We may easily determine where an *Upright* Table ought to be plac'd, that the same Object, may have the same Perspective thereon, which it has on any *Horizontal* Table; the Eye keeping the same Position in each Case.

As if (Fig. 32.) the Eye being at A, we were to find where a *vertical* Table ought to stand, as in *ex. gr.* where in HN or SP, *&c.* that so the Perspective of the Line DE thereon may be the same as that of the said Line DE, upon the *Horizontal* Table BE; the Eye continuing still at A.

Put HO=DE. Then by Similar Angles HO : HI :: GO : AG, also DE : CE :: DN : AN; wherefore the Perspectives CE and HI, are as $\frac{DE \times AN}{DN}$, and $\frac{HO \times AG}{GO}$, or $\frac{AN}{DN}$ and $\frac{AG}{GO}$. Therefore if CE=HI, then will AN : DN :: AG : GO, and so the Rectangular Triangles ADN, AGO are Similar, and therefore the Angle ADN, =AOG, or DAB=OAN, or DAB+DAN =OAN+DAN; but DAB+DAN, or BAN, is a Right Angle by *Construction*, and therefore OAN+DAN must be so too. And consequently we must draw

AQ,

AQ perpendicular to AD, and having ſet off QP=DE, erect the Perpendicular PS; for then on this Table, ſhall the Perſpective PR, be equal to CE, upon the Horizontal Table; the Eye in both Caſes being at A. *Q. E. I.*

COROL.

If the Figures BN and AN, were *Squares*; then in this Caſe the Vertical Table ought to ſtand in NH, in order to our having the Perſpective NH=CE.

For they being Squares, then AG=AN =EN=GH; and becauſe HO=DE by *Suppoſition*, ∴ GO=DN, and ſince CE HI :: $\frac{DE \times AN}{DN} : \frac{HO \times AG}{GO}$, 'tis plain the Scenographick Projections, on theſe two Tables are equal to each other.

Of the Practice of Perſpective, on Tables Inclin'd to the Horizon.

Though the Rules of this Perſpective, have much Affinity, with thoſe before demonſtrated for *Vertical* or *Upright* Tables; yet there is not that Sort of Coincidence, or Agreement betwixt them, that ſome of the Writers of this Science have imagin'd.

Thus

Thus (for Example) M. *Lamy*'s Account of this Matter, as we find it in his *Perspective, printed at London* 1702; is far from being either clear or genuine: and that abating all Miſtakes of the *Preſs*.

Let (Fig. 33.) BG be the Ground Plane, SE an *Upright* Table; HE an *Inclin'd* one, the Eye's Height, AB, its Diſtance from the *Vertical* Table, AP, the Line BC = AB, and ||[1] EH, and from C, a Line as CN ||[1] BG, ſtriking the Inclin'd Table in the Point N, and therefore = BE.

Suppoſe the Eye at C, viewing any Point as G in the Ground Plane, and by the Viſual Ray CG, making its Perſpective Seat, in the *Inclin'd* Table at F; ſo that EF, is the Perſpective of FG, on the ſaid Table, to the Eye at C.

Now he tells us, that (keeping the ſame Point of *Station* E) if we bring the Table EH into the *Upright* Poſition ES; and the *Spectatour* moves back from BC to BA; that then the Point C coinciding with A, and N with P, the Point F which is the Perſpective of G, to the Eye at C, upon the *Inclin'd* Table, will alſo coincide with D, which is the Perſpective of the ſame Point G, to the Eye at A, upon the *Upright* Table: that is, that EF = ED.

And indeed it is true, that if the Poſtures of the *Spectatour* and the *Table*, are thus ſhifted

ſhifted as he ſuppoſes; the Points C and A, N and P, will coincide. Alſo I allow that F and D will do ſo too; or that EF will $=$ED.

For from Sim. $\triangle^{ls}$ BG : AB :: EG : ED,
Alſo, BG : BC :: EG : EF,

Therefore ED$=$EF; tho' at the ſame time, his way of proving it, is (to ſay no more) very confus'd and odd.

However, that we paſs by; and grant him, that ED is equal to EF. And what if it be ſo; what follows from thence? Why then ſays he; *The Perſpective of G, after this Change of the Poſitions, will be found in the very ſame Point of the Picture*; that is, when the Table is ſet upright, and the Spectatour has erected himſelf likewiſe; it will be juſt were it was, when both were inclin'd. Very well! And now then, what is the Rule ariſing from hence, in order to Practice? Why he tells us, *That we are to draw the Perſpective of an Object upon the upright Table ES, according to the Rules before given; making AB* ($=$BC$=$EN) *the Height of the Eye, and its Diſtance,* AP ($=$BE$=$CN;) *the Point of Sight P, in this Table, being the ſame with N, in the other; becauſe the Lines EN and EP are equal. And when this is done, we are only to ſet the Table ES, back again into the Place EH, and the*

Spectatour

Spectatour to betake himself to his stooping Posture, so as to place his Eye in the Point C; *and then the Perspective will answer Expectation.*

But this Gentleman to be sure, did not consider, that tho' the Perspective of G, on at the *Inclin'd* Table, with respect to the Eye C; does thus coincide with the Perspectives of the same Point, on the *upright* Table, to the Eye at A; and tho' there will be (by Vertue of the same Demonstration) the like Coincidence, as to the Perspectives of any other Points, taken in *the same Line* EG; yet when he comes to take a Point, that lies in some other Line; and not in EG; *he must then of necessity shift his Eye* from C, into some other Place, in order to obtain this Coincidence of Perspectives, upon the two Tables. And this will be demonstrably evident to any one, if the Tables, which are here represented by *strait Lines* only, be but represented in their proper Dimensions as *Plane Figures.* And therefore, as many different Lines of Incidence as there are, in which the Points in the *Ground Plane*, whose Perspectives he would find, are posited: *So many several Removes and Shiftings, of the Eye* from C, must there necessarily be: that is, the Spectatour must put himself, into the same Variety of new Places and Postures; in order to have the Perspectives, of the

the Points of an *Object*, on an *Inclin'd* Table, coincident with the Perſpectives,of the ſame Points on an *upright* Table. And what an eaſie and practicable Method this would be, of drawing a piece of Perſpective on an *Inclind'd* Table; I leave it to the World to Judge.

But, which is the main thing of all; he has quite drop'd the true Problem, and ſubſtituted an other in the Room of it.

All that is of Uſe, and which is what a Man would enquire after and expect in the Solution of ſuch a Problem; is *how to draw upon an Inclin'd Table*, ex. gr. *EH*; *keeping his perpendicular Poſture AB, and his Eye, continually in the ſelf ſame Point at A*: And not, how he may *ſhift his Eye from A, into an other Poſture C*, and there have his Perſpective F, coincident with D, when the Table and Spectatour are both *ſet upright again*; and ſo to go on at this Rate, into an Infinity of Poſtures and Poſitions; which in Practice, is to do nothing at all, and therefore to preſcribe it, is to teach nothing at all. Certainly, as there are Rules for Drawing upon *Vertical* and *Horizontal Tables*, not incumber'd with any ſuch precarious Changes and Removes of the Eye; ſo the like Rules be demonſtrated for Tables *inclind'd to the Horizon*; and the Principles on both Sides are ſo near

near a-kin, that the Application is not difficult, to be made from the one to the other.

PROP. XXXVIII. THEOR.
(Fig. 34.)

The Point, to which the Perſpectives of any Parallels in the Ground Plane, converge upon Tables Inclin'd to the Horizon; is (as in thoſe which are Vertical) *determin'd by the Interſection of the Table, by a Ray paſſing from the Eye, parallel to the aforeſaid Parallels in the Ground Plane.*

Let the Eye be at A, its Height AB, the Ground Plane CHDI, the Inclin'd Table CEFD, any Parallels in the Ground Plane, CLH, DKI; the Line LK parallel to the Ground Line CD. Draw the Viſual Plane ALK, whoſe Section by the Plane of the Inclin'd Table, is MN; which is therefore the Perſpective Repreſentation of the Line KL, upon the ſaid Table.

It's evident, that when the Viſual Plane ALK, becomes parallel to the Horizon, the two Sides AL, AK, coincide with each other, and the whole Plane falls into the Right Line AP, which is parallel to the Horizon, and ſtrikes the Table in G, which is the *Point of Sight.*

Hither

Hither tis, that the Lines CM and DN, the Perſpectives of CL and DK, do converge; ſo that CG and DG, are the Perſpectives of the Parallels CH, DI, *infinitely produc'd*. All which is moſt eaſily demonſtrated by the *Priſm*, after the manner that we proceeded at *Prop*. IX. for *upright* Tables. *Q. E. D.*

PROP. XXXIX. THEOR.

The Perſpectives of all Lines parallel to the Ground Line, *are parallel one to another; upon* Inclin'd *Tables as well as* Upright *ones.*

Thus if the Line LK, be parallel to the Ground Line CD, we will demonſtrate (as at Prop. XI.) that its Perſpective MN, ſhall alſo be parallel to CD; and therefore, all parallel to one another, *Q. E. D.*

COROL.

Therefore all theſe Lines, are drawn upon *Inclin'd* Tables, after the very ſame Manner as upon thoſe that are *Vertical*.

PROP. XL. THEOR.

The Perspectives of Lines, Perpendicular *to the* Ground Plane, *are to be drawn upon* Inclin'd *Tables, after a very different Manner, from what they are on* upright *ones.*

For by Prop. XII. upon *Vertical* Tables, these Perspectives, are all *Perpendicular* to the *Ground Line*, and consequently parallel to one another.

The Reason of which is, because the *Visual Planes*, which are all perpendicular to the Horizon, being cut by the Plane of the *Table*, which is likewise perpendicular to the Horizon; their common Sections (*viz.* The Perspectives of the Lines, perpendicular to the Ground Plane) must necessarily, be all of them, perpendicular to the Ground Line of the Table.

But is can't be thus, when the Table is plac'd stooping or inclin'd to the Ground Plane. For the *Visual* Planes, which are all perpendicular to the Horizon, being cut by the Inclin'd Oblique Plane of the Table; will not make the common Sections or Perspectives, perpendicular to the *Ground Line*; but *inclining*, and that in various Angles of Obliquity.

Not

Nor is there any more, than one Section only, of the *Inclin'd* Table, by a visual Plane; wherein the *common Section*, is at right Angles to the *Ground Line*.

But *one* there is; which is, when the *Visual* Plane, cuts the Inclin'd Table at Right Angles. In this Case, the Perspective, of a Line perpendicular to the *Ground Plane*, will also be perpendicular to the *Ground Line* of the Table; otherwise not, as any Man may easily satisfie himself, from the common Principles of Geometry. *Q.E.D.*

PROP. XLI. THEOR. (Fig. 35.)

If ADBOp be the Plane of a Table, inclin'd to the Horizontal Plane DrOn; the Eye at C, its Height Cr, from whence a Perpendicular as rOn, is let fall to the Ground Line DOp; Lastly, *CB parallel to rn, or the Horizontal Plane, striking the Inclin'd Table in B: I say, that* n *being any* visible *Point in the Ground Plane; if we set off Op =On, and AB=CB, and carrying up the Line OB from O to B, join the Points A and p, with the right Line Ap, intersecting OB in* c; *that then the Point* c *shall be the true Perspective Seat, of the given Point* n, *in the Table.*

CONSTRUCTION.

From the Eye at C, draw Cs ∥[l] Bo, as also AD ∥[l] to the same Bo, from the Point A. Draw the visual Ray Cn, cutting the Table in the Point *a*.

DEMONSTRATION.

The Point *a*, is the *natural* Perspective of the Point *n*, to the Eye at C, upon the Inclin'd Table; and by Prop. XXXVIII. this Point *a* is found some where in the *Radial* Bo. Now I'll demonstrate that *a* is coincident with *c*, determin'd by the Intersection of the Lines A*p* and B*o*, drawn after the Manner, as is express'd in the Proposition.

The Triangles sCn, *oan*, are Similar;

Therefore, $sn : on :: sC : oa$.

Again, the Triangles ADp, *ocp*, are Similar;

Therefore, $Dp : op :: AD : oc$;

But $sn = so + on = CB + on$ (because of ∥[ls]) $= AB + on$ (*Construct.*) $= Do + on$ (because of ∥[ls]) $= Do + op$ (*Construct.*) therefore $sn = pD$.

Again; $on = op$ (*Construct.*)

And $sC = Bo$ (because of Parallels) $= AD$ (because of Parallels) therefore the *Fourth* Terms

Terms of the Proportions are reſpectively equal, *viz.* $oa=oc$.

Therefore the Points *a* and *c*, do coincide with each other on the Table.

That is, the Point *c*, determin'd by the Interſection of the Lines Ap and Bo, is coincident with *a*, the *natural* Perſpective of *n*, in the Ground Plane.

Therefore the Perſpective of the Point *n*, in the Ground Plane, is rightly determined, by laying off in the *Ground Line*, $op=on$, and in the *Horizontal* Line, $BA=BC$, and then drawing A*p*, to cut the *Radial* Bo in the Point *c*. *Q. E. D.*

SCHOL.

This Demonſtration for Inclin'd Tables, proceeds exactly after the ſame manner, with that general one givenat *Prop.* IX. For Tables*perpendicular* to the Horizon. All the Difference is; that the Line CB, is there *perpendicular* to the Table, and here *oblique*; which neceſſarily ariſes from the different Poſition of the Table in that Caſe and this: But in both Caſes it's parallel to the Horizon, and where it ſtrikes the Table, (as here at B) determines the *Point of Sight*, if the Lines *on*, *&c.* are at Right Angles to the Ground Line Dp; or otherwiſe, ſome *Accidental Point*.

And those that will take the Pains to draw the Figure out, may accommodate the Demonstration to any Case; let the Line *on*, in the Ground Plane, lie (as it does here) passing through the Foot of the Eye's Perpendicular Cr; or any other ways on either Side of the Eye. But indeed, the bare Inspection of the Fig. referr'd to at the forementioned Prop. will be a sufficient Proof of the Universality of this Demonstration, for *Inclin'd* Tables, without any more ado.

COROLLARY I.

Hence then we have a Method of tracing out practically, upon any Inclin'd Table, the Perspective Seats, of any given Point or Points, in the Ground Plane; and consequently of delineating the entire *Scenographick* Appearance of any Object, upon such a Table.

COROL. II.

The Distance BA in the Horizontal Line (which determines A, the *Succedaneous Point of Distance*) being $=$CB$=os=sr+ro$; is therefore $=$ the *perpendicular Distance of the Spectator from the Ground Line*, added to the *Cotangent of the Tables Inclination*;

tion, the Eye's Height being the Radius. For the Angle *Csr* = *Bon*, the Tables Horizontal Inclination.

Alſo. oB, the Perpendicular Diſtance, of the *Point of Sight*, above the Ground Line D*p*; is the Co-Secant of the ſame Angle: Which Remarks may be uſeful in Practice.

PROP. XLII. PROB. XVI.

To draw upon an Inclin'd *Table, the Perſpective of any Line,* perpendicular *to the Ground Plane.*

How all Sorts of Lines *lying* in the Ground Plane, are to be drawn in Perſpective upon theſe Tables; we have ſhewn already: But how thoſe which are *raiſed above* the Ground Plane, are to be repreſented, is a thing of more Difficulty. I ſhall ſhew therefore, how we may eaſily and practically trace out upon a Table, any how inclin'd to the Horizon, the Perſpe-ſpective, of any Line, which ſtands *erect on the Ground Plane*: it being eaſie from thence, to draw the Perſpectives of any Lines that are *oblique* thereto; as we have intimated before, in *vertical* Perſpective.

From the Foot of the *Perpendicular*, where it ſtands in the *Ground Plane*, carry a Line of *Incidence* to the *Ground Line* of the Table.

On this Line of Incidence, imagine a Plane to be erected perpendicular to the *Ground Plane*; by which Means, it will alſo be perpendicular to the *Inclin'd Table*; and its *common Section therewith*, will be at Right Angles to the Ground Line. This common Section, for Diſtinction ſake, I call *the Perpendicular of the Table*, and is repreſented in Fig. 35. by the Line B*o*. Thro' the Apex or Top of the Perpendicular, conceive another Plane to be carried, *parallel* to the Ground Plane, whoſe common Section with the Table, will be *a new Ground Line*, and parallel to the former below.

Where the common Section of this *Horizontal* Plane, and the former *Erect* one, cuts the Table, will be the *new Point of Incidence*, for the Apex of the Perpendicular; whoſe *Diſtance* alſo, from the *new Ground Line*, is that Part of the aforeſaid common Section of the two Planes, which lies between the *new Point of Incidence*, and the Apex of the Perpendicular.

And thus, having the two *Points of Incidence*, *viz.* for the Foot, and Apex of the Perpendicular; as alſo the *Diſtance* of each

each from its reſpective Ground Line; we are only to determine (by the Help of the Rule demonſtrated in the laſt Prop.) the *Perſpective Seat* of each of theſe Points) and ſo join them with a Right Line; which will be the true *Scenographick Appearance, of the Perpendicular propoſed*, upon the Inclining Table.

Now the *Inclination* of the Table, the *Height* of the propos'd Perpendicular, and the *Diſtance* of its *Foot*, from the Ground Line, being all actually given; it's eaſie to find, the Diſtance of the Apex, from the *new Ground Line*, and whereabouts the *new Point of Incidence* falls in the Table.

For; *Radius*, to *Co-ſecant* of the Tables Inclination, ſo the given *Height of the Perpendicular*, to a Fourth; which is equal to the Segment of *the Perpendicular of the Table*, intercepted between the Points of Incidence, of the Foot and Apex of the Height propos'd.

And as *Radius*, to *Cotangent* of the Tables Inclination, ſo the *given Height*, to a 4th; which ſubtracted from the Diſtance of the *Foot* from the Ground Line; gives the Diſtance of the *Apex* from the new Ground Line.

And after this, I believe there cannot be much Difficulty remaining, with Reſpect to the Practice of Perſpective upon theſe ſort of Tables.

The Inverse Method of Perspective.

Hitherto we have been conversant, about that Part of the Practice of Perspective, which is very properly call'd the *Direct*; since it is the Method of proceeding, from the Object it self, to its Perspective Appearance; so that knowing the true Form and Position of the *Former*, we can immediately trace out the *Latter* on the Table.

The *Inverse* of this, shews how, by a retrograde Sort of Process, from the Perspective given; to determine the Figure and Situation of the *Original* or Prototype: Which Method I shall now exemplifie in some few Problems, but sufficient to lead the Reader (who is well instructed in the foregoing Practice) into all the Parts and Steps of this.

PROP. XLIII. PROB. XVII.

Any Point in the Perspective Table, being given; let it be required, to find its Original Seat in the Ground Plane. (See Fig. 15. N°. 2, 3.

Let O be a Point given in the Table; thro' which draw any two Lines at Liberty, which produce, till they cut the *Horizontal Line*, *ex. gr.* in G, and D, and

and the *ground Line* in N and M; ſo that the entire Lines themſelves, are DOM, and GON. From the Points D, G, let fall the Perpendiculars DF, GH, to cut the Ground Line in the Points F, H; and from B, draw the Lines BF, BH. Then from the Points M, N, draw the Lines MK, NL, *parallel* to BF, BH, reſpectively; which interſect each other in I: I ſay, that I is the true *Original*, of the Point O in the Perſpective Table.

Otherwiſe thus. (Fig. 15. N°. 1.)

Let *f* be any Point in the Table, the Seat of whoſe *Original*, in the *Ground Plane*, is required.

Draw a Line from B the *Point of Sight*, thro' the given Point *f*, till it cut the Ground Line in D; at D, erect DF perpendicular to the Ground Line; and having drawn a Line from the *Point of Diſtance* C, thro' *f*; produce it till it cuts the Ground Line, in E. In the Perpendicular DF, Set off DF=DE. Then will F be the Point ſought.

The Demonſtration of theſe Practices will be very eaſie, to thoſe that underſtand the Reaſon, of the Operations at Prob. I.

However, for the ſake of thoſe, that may deſire to ſee them demonſtrated, I ſhall add the Demonſtrations, and that of each Practice diſtinctly. DEMON-

DEMON. *of the First Practice.*

The Lines OG and OD, are drawn from the Point O, till they cut the *Horizontal* Line in G and D. And (by *Construction*) the Lines GH and DF, are Perpendiculars from the Points G and D, to the *Ground Line* EF.

Now by *Corol.* VI, VII. *Prop.* IX. all Lines in the *Ground Plane*, parallel to BF, run up in the Perſpective Table, to the Point D; as alſo theſe which are parallel to BH, converge to G.

But (by *Conſtruct.*) the Lines NL and MK, are reſpectively parallel to BH and BF.

Therefore the Lines NL and MK in the *Ground Plane*, infinitely produc'd; are the *Originals* anſwering to the Perſpectives NG and MD.

Therefore the Point I, which is the Interſection, of the ſaid infinitely produc'd Lines NL and MK, is alſo the true *Original*, of the Point O in the Perſpective Table. *Q. E. D.*

DEMON. *of the Second Practice.*

By *Conſtruct.* the Line DF is perpendicular to the *ground Line*, therefore by *Corol.*

Corol. VI. *Prop.* IX. the Line DB, is the Perſpective of DF infinitely produc'd; or *vice verſâ*, DF infinitely produc'd, the Original of BfD; and conſequently the *Original* of the Point *f*, muſt of Neceſſity be ſomewhere in the ſaid Line DF produc'd. But alſo ſince C is by *Hypoth.* the Point of *Diſtance*, and the Points C, f, E, do by *Conſtruct.* lie all in a Right Line; and moreover ſince by *Conſtruct.* DF is taken equal to DE; therefore by *Prop.* X. F is the true *Original* of the Point *f* in the Table. *Q. E. D.*

PROP. XLIV. PROB. XVIII.

Giving any Line in the Table, *to determine its* Original, *in the Ground Plane.*

1. Let the Line given in the Table, be terminated both ways, *viz.* by the *Horizontal* and the *Ground Line*; *ex. gr.* MD.

From the Point D in the Horizontal Line, let fall DF perpendicular to the Ground Line, and interſecting it in F; then from B, the Foot of the Eye's Perpendicular, draw a Line to F; and from M (where the Line given cuts the *ground Line*) draw MK at Liberty, *parallel* to BF. I ſay that the Line MK infinitely produc'd, is the true *Original or Prototype* of MD in the Table. The

The Demonſtration is apparent, from what has been ſaid before.

2. Let the Line be terminated by the *ground Line*, and ſome other Point in the Table; as *ex. gr.* the Line MO.

Having produced the given Line MO, till it cuts the Horizontal Line of the Table in D, and drawn DF, and BF, and MK, in that manner that was ſhewn juſt now; we have nothing to do, but only according to *Prob.* XVII. foregoing, to determine the *Original* of the Point O, or what Point in the *ground Plane*, belongs to O in the Table. And having by that Means found the Point I, we have conſequently, the Line MI, for the Prototype of MO.

3. Let the Line be terminated by the *Horizontal Line*, and ſome other Point in the Table; as *ex. gr.* the Line DO.

Produce DO, till it cuts the Ground Line in M, and draw DF, BF, MK, as before; then determining I (in MK) for the *Prototype* of O; all the infinite Production of the Line IK beyond I, will be the true *Original* of DO.

4. Let the Line given in the Table, be terminated neither by the *Horizontal*, nor the *ground Line*; as *ex. gr.* the Line PQ.

Having

Having produc'd it both ways, till it cuts the *Horizontal Line* in C, and the *ground Line* in R; from C, let fall CE perpendicular to the ground Line, and draw BE; then from R, extend the Line RL at liberty, parallel to BE; in which Line, the Points K, L, the respective *Originals* of Q, P, may be determin'd (as by *Prob.* XVII.) and consequently KL, for the *Prototype* of the given Line PQ.

Or thus: Having drawn CE, BE, and RL, as before; take any Point in the Table, at Liberty, as O, and from thence carry two Lines at Liberty, thro' the Extremities P and Q, producing them till they cut the Ground Line in M,N, and the Horizontal Line in D, G; from whence let fall the Perpendiculars DF, GH, and from B, draw the Lines BF, BH. Then from the Points M, N, draw the Lines MK, NL, at Liberty, parallel to BF, BH respectively, which produce till they intersect the Line RL, in the Points KL. I say, the Line KL is the true Original of QP.

PROP.

PROP. XLV. PROB. XIX.

Any Angle *being given in the* Table; *to find an Angle in the* Ground Plane, *to which the said Angle in the Table, is equal in Representation.*

Suppose the Point *P*, was taken at Liberty in the Table, through which were drawn the Lines PO, PQ, any how, so that the Angle OPQ be formed; and it be requir'd to determine the Angle in the Ground Plane, to which the said Angle OPQ is equal in *Representation*.

The Lines containing the given Angle, being produc'd till they cut the *Horizontal* Line, in the Points G, C, and the *ground Line* in N, R; from the Points G, C, let fall the Perpendiculars GH, CE, each equal to the Eye's Perpendicular AB. From B draw the Lines BH, BE, to the Points H and E; and from the Points N and R before determin'd in the *ground* Line, produce the Lines NI and RK, parallel respectively to BH, BE, which intersect each other in the Point L. I say that the Angle RLN, is the Angle sought, *viz.* That to which OPQ in the Table, is equal in Representation.

For

For from the Practices already demonstrated, we will ſhew, that the *Prototypes* of PO, and PQ, are found in the Lines NI, RK produc'd, and conſequently that the Angle of the *Prototypes* NLR, is the true Angle repreſented by OPQ in the Table.

S C H O L.

After what has been demonſtrated of the Practice of the *Inverſe* Method of Perſpective, with reſpect to *Lines* and *Angles*; there can be no Difficulty remaining, how to extend the ſame to *Plane*-Figures, or even to *Solids* themſelves. 'Tis true, that will be more laborious; however, there are no new Rules, and 'twill be but a bare Repetition of the Work already done. *Ex. Gr.* If we were to compleat in the *Ground-Plane*, the *Original* of the Plane Figure OPQ in the *Table*.

Having determin'd the Angle NLR, anſwering to OPQ in the *Table*, find another Angle in the Ground Plane, equal in Repreſentation to ſome other in the Table, *ex. gr:* to O, or Q.

I'll take O for Example; and producing QO, till it cuts above and below in D and M, and drawing DF and BF as before; I produce MI, parallel to BF, which interſects the Lines RL and NL, before drawn,

in I and K, and ſo cuts off the Figure IKL, for the true *Prototype* of OQP.

And thus we have gone through all that is of grand uſe, either in the *Direct* or *Inverſe* Practice of Perſpective; and I'll venture to ſay, That one who well underſtands the foregoing Practices, with their Demonſtrations, may ſucceſſively attempt any Problems whatſoever relating to either of them.

I ſhall conclude, with ſome brief Obſervations, upon *a moſt curious and uſeful Problem* in this Science, which, to the beſt of my Knowledge, has never been ſo much as touched upon, by any of thoſe who have written the moſt *Mathematically* this way; for nothing of this Kind, is ever to be expected from the common Mechanical Practitioners.

The Problem is this, *viz.* Giving an *Object* in the Ground Plane, with its *Diſtance* from the Table, and the *Height* of the Eye: To find ſuch a proper *Diſtance of the Eye* from the Table, that the *Original* or *Prototype* may be to its Perſpective on the Table (Area to Area) in any given Ratio of *Majority*.

Note, I ſay, in any given Ratio of *Majority*: For the Section of the *Viſual Pyramid* on the Table, will ever be leſs, than its Baſis, in the Ground-Plane. To

To ſolve this Problem, in one of the moſt uſeful Caſes in Practice; will be as much as I need do: Thoſe that have a mind to do it in more, may do it at their Leaſure.

And though I have actually ſolv'd ſome Caſes of this Problem by *Algebraically*, yet I ſhall not bring in thoſe Computations here; being reſolved to uſe no other, but pure *Geometrical* Reaſonings in this Treatiſe.

Suppoſe therefore (at Fig. 6.) that the Object DE, being a Circle in the Ground Plane, it were requir'd that its Perſpective upon the Table GD, ſhould be a Circle, and a Circle, whoſe Area ſhould be to that of DE, in the Ratio of the Line N to the Line M.

The *Height* of the Eye is ſuppoſed to be *given*, which therefore we will denote by the Line H.

But what is wanted, is that particular *Diſtance* of the Eye from the Table; that the viſual Cone may not only be cut *ſubcontrarily*, by the Plane of the Table; but alſo that the *Section* may be to the *Baſis*, in the aſſigned Proportion, of N to M.

Without ſuppoſing any thing at all of the *Circle*, or the *Conſtruction* mention'd before at that Fig 6; we'll imagine the Eye to be at the Point F; being wholly ignorant

whereabout the Point F is, or how far distant from the Table. And therefore imagining the Visual Rayes FE, FD, to be drawn, we'll suppose the Section sought for, to be CD.

Now ⊙ DE : ⊙ DC :: M : N (*Hypothesis.*)
That is ; $DE^q : DC^q :: M : N$.
But $DE^q : DC^q :: FE^q : FD^q$ (*Subcontr. Section.*)

And supposing a Perpendicular from the Eye to the *ground Plane, viz.* FB; a Line = FD, will fall some where on the other Side of it. Let that Line be FA.

Then $DE^q : DC^q :: FE^q : FA^q$.

Because BF (wherever it falls) is perpendicular to the Ground Plane, therefore it is parallel to the Table GD.

Therefore DFB = FDC.
But FDC = FED (*Subcontr. Sect.*)
Therefore DFB = FED.

But because FA = FD (*Construct.*) and FB is perpendicular (*Construct.*) therefore AFB=DFB, and AB=BD.

Therefore AFB=FED.

Therefore the Angle AFE, must be a *Right* one.

If

If ſo, then it muſt be $FE^q : FA^q ::$ $BE : BA$.

But $FE^q : FA^q :: M : N$ (*Conſtruct.*)
Therefore $BE : BA :: M : N$.
But $BA = BD$;
Therefore $BE : BD :: M : N$.
And $BE - BD : BD :: M - N ; N$;
That is $DE : BD :: M - N : N$.

Therefore BD the proper Diſtance of the Eye from the Table, is determin'd. *Q E F.*

AN

APPENIDX

CONTAINING

A brief Account of ſome Things, of Uſe, in the ART

OF

PERSPECTIVE.

AN APPENDIX.

I. *Of Scenes for the Stage.*

A *THEATRE* painted according to the Rules of Art, appears a Regular Piece of Perſpective; when viewed from a certain Point. Nor are there any other Rules needful to the Underſtanding, all the *Mathematical* Part of this fine Piece of Theory; than only ſome of thoſe demonſtrated in the foregoing Treatiſe.

I believe there is none that has written, both ſo much, and ſo curiouſly upon this Subject, as the Famous *Jeſuit Andrea Pozzo*, in his two Volumes of *Architecture in Perſpective*, eſpecially the *Second*; which therefore, all that are deſirous of being informed in theſe Matters, will do very well to conſult. But there is enough in the *First* Volume, to let any Perſon, into the true *Knowledge*, *Uſe* and *Conſtruction* of

of Scenes, who, together with theſe Rules, actually ſees the Diſpoſition of all in a Theatre.

Here the Reader may find, the proper *Dimenſions* of the ſeveral Parts of a Theater; the Method of finding the *Point of Sight* therein, and the *Diſtance* from whence it ſhall appear a Juſt Piece of Perſpective. Alſo how the Scenes are diſpos'd and directed in their *Grooves*; what their proper Heights are; how by knowing the Width between the *neareſt* and *fartheſt* Grooves, the *Length* of the Theatre, or the Diſtance of its *Point*, from the Edge of the *Stage*, may be found; with various other curious and uſeful Practices relating to this Matter.

All which being ſo amply and particularly treated of, by the aforeſaid Author; I ſhall not need to enlarge on them here, but refer the inquiſitive Reader thither, where he will meet with all the Satisfaction he can reaſonably deſire.

II. *Whether more than one Point of Sight only, be to be admitted in Pieces of Perſpective.*

To anſwer this, 'tis neceſſary that we diſtinguiſh, with the *Excellent Author* juſt now mention'd.

'Tis

'Tis one thing to ſpeak *in the Groſs*, of any Work of large and great Extent; and another thing, to ſpeak of the *ſeveral diſtinct Parts of that Work*.

In the *former* Senſe, more Points of Sight, than one, are to be admitted.

In the *latter*, *viz.* When we ſpeak of *any particular Part of a Work*, we ought to aſſign but one Point of Sight only; and to each ſeveral Part, its own proper Point.

Many Points of Sight introduc'd into the ſame Piece, or where there is one ſole entire Deſign; would be more prejudicial to the Work, in many Reſpects, than the making uſe of one *only*.

For whereas, if one only be made uſe of, 'tis then plain, that from ſome *one determin'd Point*, a compleat and *perfect View*, may be taken of the whole Peice; if ſeveral be introduc'd, then there is no one Point, from whence you can have a perfect View of the whole; but all that can be done, is to view each ſeveral Part of it, from its own proper Point.

Beſides, good *Painting*, being but an Imitation of Nature; a Painter is not oblig'd to make his Work appear real, or as the very *Life*, from any Point; but from *ſome determinate* Point only. Nor indeed is it poſſible that he ſhould do ſo. For if *this Picture*, *ex.gr.* upon ſuch a *Table*, be

an

an exact Representation of the *Life*; the *Life* it self, and the *Eye* which draws it, being in this or that *Position*; 'tis impossible that that same Picture, should be an exact Representation of the same Life, as it appears to that Eye, which is now shifted into a new Place or Position.

In a Word; together with the Reason of the thing, we may add this also; that one Point of Sight only, is to be found in *the Performances of the greatest Masters*, when 'tis a simple Design, and the Work consists but of one Piece,

III. *How to avoid Confusion, in setting Plans or Elevations, in Perspective.*

If when the *Plan* of any Figure is drawn in Perspective, it happens (thro' the *too great Obliquity* of the *Visual* Lines) that the Parts of it are crouded too close together, and by that Means become *confused* and indistinct; this may be easily remedied, by making choice of a new *Ground Line*, farther distant from the *Horizontal Line*, and so drawing a fresh Perspective Plan; which if not yet distinct enough, the Space between the Horizon and the Ground Line may be still enlarg'd, and so a new Plan drawn as before,

So

So likewiſe in *Elevations*, when by Reaſon of *too near an Approach to the Point of Sight*, the Projectures of the ſeveral Altitudes, cannot well be diſtinguiſh'd and deſign'd; the Remedy for this Inconvenience, is by ſetting the Elevation at ſome due Diſtance, *farther* from the Point of Sight; in which Caſe, the Parts which before were confuſed, by Reaſon of the too great Obliquity, will now become more obvious and diſtinct.

IV. *How deficient Figures, may be made to appear compleat, or any Figures may be made to appear of other Dimenſions, than they really are; by the Help of Perſpective.*

'Tis ſuppoſed here, that ſome certain determinate Point is fix'd, from whence 'tis requir'd, that the Figures ſhould appear compleat.

And the Perſpective Work being done with Reſpect to that one Point; it's impoſſible it ſhould appear perfect, when view'd from any other Point but that alone.

Suppoſe a *Room* were of ſuch a Figure, as wanted only one Angle or *Corner*, of a *true Square*, and this Defect were to be remedied by Perſpective; and the Room which is now a Trapezium, were to be

made

made appear from a certain Point as it would, if it had been really Square.

If the *deficient Triangle*, be painted on the *Wall*, adjoyning thereto, as it ought to be by the Rules of Perſpective, for the Eye in the Point aſſigned; I ſay then, that if the Place be viewed from that Point, it will appear, as if it had been a true Square.

If any Space were to be made appear *Longer* or *Broader*, than it really is, according to any Meaſure or Proportion aſſigned; this will be done by drawing according to the Rules of *vertical* Perſpective, on the *Wall* or *Wainſcoat*, at the fartheſt End, that Augmentation of the Area which is required.

As for Example, if an Area or Groundfloor were 40 Foot long and 10 broad; and it were to be made appear (keeping the ſame Width) as an Area of 60 Foot long: I ſay, that if an Area of 20 Foot, be painted in Perſpective on the Wall or Wanſcoat (as a *Table*) according to the *proper Diſtance*, from whence it is to be view'd; that then to an Eye fix'd in that Point, it will appear as if the Space it ſelf were in reality ſo much longer.

The Reaſon of which is moſt obvious; for ſince by the Suppoſition, the *deſired Increaſe of the Area*, is drawn in Perſpective, according to the due Height and Diſtance

ſtance of the Eye, on a Table erected at the fartheſt End of the Space propos'd; that Piece of Perſpective will undoubtedly appear to the Eye ſeated in its proper Point, juſt as the deſired Prolongation of the Area it ſelf would have done, if it had been true and real. And conſequently, the whole together, *viz.* the real Area of 40 Foot, and the Perſpective of the 20, upon the Wall; will appear, as an Area, of the real Dimenſions of 60 would have done.

If the ſeeming Increaſe of Dimenſions were to be *upwards*, inſtead of *long* or *broadways*; as if for Example, a *Room* or *Entry*, were to be made appear *higher* than it really is: In order to this Effect, we are to conſider the *Roof* or *Cieling*, as a Table, and thereon to draw by the Rules of *Horizontal Perſpective*, the Repreſentation of what we would have, according to the intended Place and Seat of the Eye. Nor can there be any manner of Difficulty in any Practices of this kind, when the Agreement between *Horizontal* and *Vertical* Perſpective, which we have demonſtrated at *Prop.* XXXVII. is well conſider'd.

By ſuch like Artifices likewiſe (tho' not quite ſo eaſily, becauſe it is more difficult to draw Pieces of Perſpective accurately, upon *Inclin'd* Tables, than upon *Upright*

Ones)

Ones) may *Inclining or Reclining Walls*, be made to appear Erect; and by that painting on the said Walls, the true Scenographick Appearances, of those Parts of the Areas, of the Roof or Floor, which are deficient or redundant, from, or above, what would be taken in, by a true Perpendicular Position. For when a Wall *Inclines*, the under Pavement or Floor, is greater than the Cieling; when it *Reclines*, 'tis less; when *Erect*, both are equal.

V. *Of Lights and Shaddows.*

Besides the rigorous *Mathematical* Part of Perspective, which shews upon demonstrative Principles, after what Manner the *Outlines* of Bodies are to be represented, or drawn upon a Table, for any given Height or Distance of the Eye; there is another, relating to the *apt Distinction of Lights and Shaddows:* Which depending much upon *Nature and Observation*, one may call (not improperly) the *Physical* Part of Perspective. 'Tis the Perfection of this Skill, that mainly surprizes in all Performances of this Kind. A just Mixture of Lights and Shaddows, without *accurate designing*, would not indeed please a *Judge* in these Matters; but the best *Design* in the World, with unnatural Lights and Shaddows, would

would ſcarce pleaſe any Body. It's leſs eaſie for a Fault (that is any thing groſs) to lie concealed in *this Part*, than it is in the *other*; which is concern'd only in the drawing the Projection of Lines. For a Fault here, is a Fault againſt *ſenſible Nature*, which every one that obſerves, is a Judge of; but there, it is againſt *Mathematical Demonſtration*, which few are converſant with.

How ſhocking would it be, to ſee in a Picture, *a very, deep and ſtrong Shaddow, together with a dark and cloudy Skie*: Or *the Lights let in, on the ſame Side, that the Shaddows fall of*; when every Body knows, that the Light muſt neceſſarily come on the contrary Side!

Theſe are Blunders which would eaſily be corrected by thoſe, that might not be able to tell, *whether ſuch and ſuch Lines were rightly directed to their proper Point in the Picture, or no*; *or whether ſuch a Column, or Tree, were aptly diſpos'd, and of its juſt Height and Form.*

However, it may be uſeful to obſerve in theſe Caſes, that beſides the *Part or Quarter*, from whence the Light comes; the *Quality and Kind*, the *Altitude and Diſtance* of the Lights, and the *Manner* of the Illumination, is to be regarded.

It's one thing, to repreſent Bodies as enlightned by *Torches or Candles*, and another, as by the *Sun*.

Again, Things that are in the *open Air*, are not enlightned after the ſame Manner, as thoſe that receive the Light, only thro' a *Hole or Window*.

Nor is an Object enlightned by *ſeveral* Luminaries, after the ſame manner, that it is by *one alone*. Nor when it is enlightned by a *Reflex'd or Reverberated Light*, is it to appear with that lively Brightneſs; as when it is expos'd to *open* and *direct* Radiations.

All theſe are to be conſider'd, before a Man attempts to expreſs the Effects of Illumination in a Picture. It's certain, for Example; that the *nearer* the Lucid Body is, by ſo much the more *Divergency* have the Rays of Light; and the *farther*, by ſo much the more do they approach, to a ſenſible *Paralleliſme*. And therefore Illuminations by *Lamps*, and ſuch like *very near* Luminaries, are to be expreſs'd *Scenographically*; in ſuch a Manner, that the *Shades* ſhould be more plentiful, than the Lights. Whereas thoſe which are caus'd by the *direct Rayes of the Sun*, are to be deſign'd *Orthographically*, ſo as that the Lights and Shades ſhould be equally diſtributed about. In like Manner, ſhould the *Poſitions*, *Altitudes*

titudes and *Distances* of Luminaries, be consider'd, in Order to give Shaddows, their due *Form*, *Proportions and Dimensions*. Not forgetting the Effects of *several* Lights conjunctly illuminating a Body, whereby the Shaddow becomes *fainter and more dilute*, than when it is projected by one single Light alone; except where the Shaddows happen to intersect, one another; for there, *viz.* at the *common Section*, the Shade is always intense and deep.

Shaddows are of no less Use, in all the *Arts of Imagery and Representation*; than they are to the Purposes of *Astronomy* and *Geography*; where they serve to evince some of the most important Conclusions in both Sciences.

'Tis by *these* that we are sometimes led into Delusions, that are infinitely pleasant and agreeable to us. We mistake a little *Paint*, for *Life* and *Reality*; think a *flat Superficies*, sometimes to be a raised solid Body, and at other times to be hollow and depressed.

And so very fine and artful, are some Impostures of this Kind, that 'tis almost impossible for the best Judges to find them out; *pure Judgment* without particular *Acquaintance and Experience*, being not sufficient to correct the Errors, we run into upon such Appearances.

It won't be amiſs to take Notice here, that *there are ſome Caſes, wherein, without particular Conſideration and Regard, had to the falling, both of the Lights and Shaddows; we are forc'd to remain in abſolute Suſpence about the true Form and Figure of a Body, whether it be Concave or Convex; really hollow and ſunk in, or elſe elevated and Protuberant:* And this upon the Score of a real Ambiguity that there is in the Appearance; ſince the Body, which is thus repreſented, may be either Concave or Convex; and it is to be determin'd only from the Lights and Shaddows, which of the two it is.

Thus for Example; ſuppoſe there is a *Round* drawn, and *ſhaded* on one Side.

I am ſure by the *Shade*, that it cannot poſſibly be a *Flat*, or a meer Circular Area, which is thus repreſented, but a *Solid*; but then whether it be *Hollow* or *Gibbous*, I cannot yet tell, without farther Conſideration. But obſerving how the *Shaddow* is drawn, and at the ſame time knowing on which Side the *Light* falls; I can eaſily determine the Matter.

For if the Light falls on the Right Side (for Example) and the Picture be ſhaded *on that* ſame *Side*; I know then, that it muſt neceſſarily repreſent a *Concave*: But if the Shade be on the *oppoſite Side*, to that on which the Light comes; I am ſure it muſt expreſs

expreſs a *Convex*. For thus the Appearances would be, if a Solid, and a Hollow *Hemiſphere*, were to be expos'd to the Light: The Protuberance of the *former*, would make it enlightened, and the ſinking in of the *latter*, would make it ſhaded; on the *ſame* Side that the Light falls.

I need ſay nothing here, how the Figures, of the Shaddows of Bodies, are to be determin'd *Mathematically*. For I have ſhewn already, at *Schol.* II. *Prop.* XXXIV. that this Practice, is only the *Inverſe*, of the Ordinary Perſpective, and may be perform'd by the Rules.

However, thoſe that pleaſe, may make uſe of the common Method; by *drawing Lines from the Light, and from the Foot of the Perpendicular, let fall from the Sight, to the Ground Plane*; which is in Effect the very ſame Method ſtill.

Thus at Fig. VII. Prop. VII. If the Rectangle APBN, were an *Opake* Body, whoſe Shaddow were to be determin'd; the Light being at K, and its Altitude KV. The Lines KA, KB, extended from the Light, thro' the Angles A, B, and produced to meet in D, E, the Lines VP, VN, drawn from V, thro' the Angles P, N, in the Ground Plane; determine the *Shaddow* PDNE: Which is alſo a Piece of ordinary Perſpective, in which DP, EN,

are *Radial* Lines running up to the *Point of Sight* V; as we have demonſtrated at Prop. XXXIV.

VI. *Of Pictures in Pictures.*

Whenever any Picture is repreſented as drawn in another Picture; the Repreſentation ought to be, according to the View of the real Spectator, who ſees the *firſt* Picture with that *ſecond* Picture in it; and not according to the View of any Perſon drawn, in the *firſt* Picture, who is imagin'd to be a Spectator of the *ſecond.*

Thus for Example, ſuppoſe a Perſon were to paint *Apelles*, drawing the Picture of *Alexander the Great.* He ought not in his Peice, to expreſs the Picture of *Alexander*, as *Alexander* appear'd to *Apelles*, or according to *Apelles*'s Veiw; but according to his own proper Image or Idea.

And the Reaſon is plain. For that Image of *Alexander*, which is a *Copy* to *Apelles*; is an *Original* or *Archetype*, to *our Painter:* And conſequently ought to be expreſs'd by him, according to his own Idea.

And from hence a Judgment may be made (by thoſe that are skilful this way) of the Defects or Perfections of many pompous Peices, wherein Repreſentations of this Kind are made. And

And thus I have ſaid, what I propoſe to ſay of theſe Matters here.

I had deſign'd in this Treatiſe likewiſe, a particular Account of *Military Perſpective*, or that which is made uſe of in the deſigning of *Fortifications*: But *this Sort of Perſpective*, depending upon quite different Principles, from what the *Common* does; ought to be explain'd and illuſtrated, with Examples, by it ſelf; which perhaps it may be, in another Place.

I ſhall here, at the Cloſe of what has hitherto been ſaid, of *Direct* Viſion, ſubjoin one Problem, relating to *Refracted* and *Reflected* Viſion.

I have formerly ſhown in another Place, [*viz. INSTITUT. FLUX.* Prob. 15.] how the *Foci* may be determin'd, for all Sorts of Glaſſes, of what Figure ſoever they are; receiving, either *parallel*, *Diverging* or *Converging* Rayes; and that by the Help of *one General Equation*, to be interpreted according to the particular Nature of each Curve: Which Method, I have ſince improv'd, in more Reſpects than *one*.

But what I intend here, is of a quite different Nature, *viz.* a *Geometrical Construction*; or an eaſie and accurate Method, of *tracing the Progreſs of a Ray by Scale and Compaſs*; and which I think to be different, from

from what I have ſeen, relating to this Matter.

Let MN (*Fig.* 36.) be ſome Refracting *Circular* Surface, *denſer* than the ambient Medium; G the *radiating* Point; GAH the Axis produc'd thro' the Centre at Liberty; GNDP, the Incident Ray, produc'd at Liberty; AN drawn from the Centre A, to the Point of Incidence N.

Upon N, with the *ſame* Radius NA, ſtrike an Arch AC; and from A, let fall AD, *perpendicular* to the Incident Ray ND, which produce till it cuts the Circle again in C; and draw NC.

The Quantities *m*, *n*, denoting the Proportion of the Sines of the Angle of *Incidence* and the *Refracted* Angle; make NP : NC : : *m*–*n* : *n*, and join PC. Let NQ *Biſect* the Angle PNC, and cut PC in Q; from whence let fall QB, *perpendicular* to DC.

Taking the *Length* of BC, in the Compaſſes, ſtrike an Arch therewith upon the Centre of the Refracting Surface A.

Laſtly, upon AN, as a Diameter, deſcribe a Semi-Circle, cutting the forementionꝰd Arch in F.

I ſay, that laying a Ruler from N to F; it ſhall cut the *Axis*, in the Point H, which is the *Focus* ſought: Or which is the ſame thing, the Line NF produc'd, is the true *Refracted* Ray.

The

The *Demonstration* of which *Construction*, is as follows.

Because AD is *perpendicular* to GN in D, therefore CND = AND = the Angle of Incidence; and so CD is the *Sine* thereof.

Farther, because PN : NC :: *m*–*n* : *n*, and the Angle PNC is bisected; therefore also PQ : QC :: *m*–*n* : *n*.

But since QB is perpendicular to AC, from thence NP and BQ are parallel;

Therefore PQ : QC :: DB : BC,
Therefore DB : BC :: *m*–*n* : *n*,
Therefore DC : BC :: *m* : *n*,
Therefore CB is = the *Sine* of the *Refracted* Angle.

Farther; because the Point F is determined by the Intersection of the Circle AFN, with the Circle whose Radius is = BC; therefore it's evident, not only that AF = BC, but also that the Angle AFN is a *Right* one, or AF perpendicular to FN.

Therefore NF is the true *Refracted Ray*, and H the Focus. *Q. E. D.*

Those that have a Mind to it, may investigate the Focal Distance, after the following or some such like Manner; having the Angle of Incidence, the Radius

dius of the Surface; the Distance of the Radiating Point in the *Axis* from the Vertex of the Surface, and the Ratio of Refraction, all given.

Imagine (*it being omitted in the Figure*) a Perpendicular, let fall from the Point of *Incidence* N, to the Axis in R.

In the Right Angled $\triangle^{l}$ AND, there's AN and AND, given; whence AD and ND are given. Also MG being given, $\therefore$ AG, and GD $(=\sqrt{AG^q - AD^q})$ are given. $\therefore$ also GN $(= GD - ND = \sqrt{AG^q - AD^q} - \sqrt{AN^q - AD^q}$ is given. By Similar $\triangle^{ls}$ GA : AD :: GN : RN, which is therefore given. Again, by Similar $\triangle^{ls}$ GA : GD :: GN : GR, which is therefore given. Therefore, GA—GR= AR, is given also.

Lastly, The $\triangle^{ls}$ RNH, AFH are Similar, therefore NR : RH :: AF : HF, that is, $NR : RA + AH :: AF : \sqrt{AH^q - AF^q}$, whence $NR^q \times AH^q - NR^q \times AF^q = AF^q \times RA^q + AF^q \times 2RA \times AH + AF^q \times AH^q$, which gives but an *Adfected Quadratick Equation*, for the finding of AH the Distance of the *Focus*, from the Centre of the Refracting Surface. *Q. E. I.*

Note 1. That this *Construction and Investigation*, suppose nothing, of *the Rayes falling near the Axis of the Refracting Surface,*

face, but proceed all one, whether nearer or further off.

2. The Refracted Ray becomes either *parallel* (to the Axis) or *Converging*, or *Diverging*, that is, the Focus, is at an *Infinite*, *Finite*, or *more than Infinite* Distance, according as the Angle EAF, is *equal*, *more*, or *less*, than a Right one.

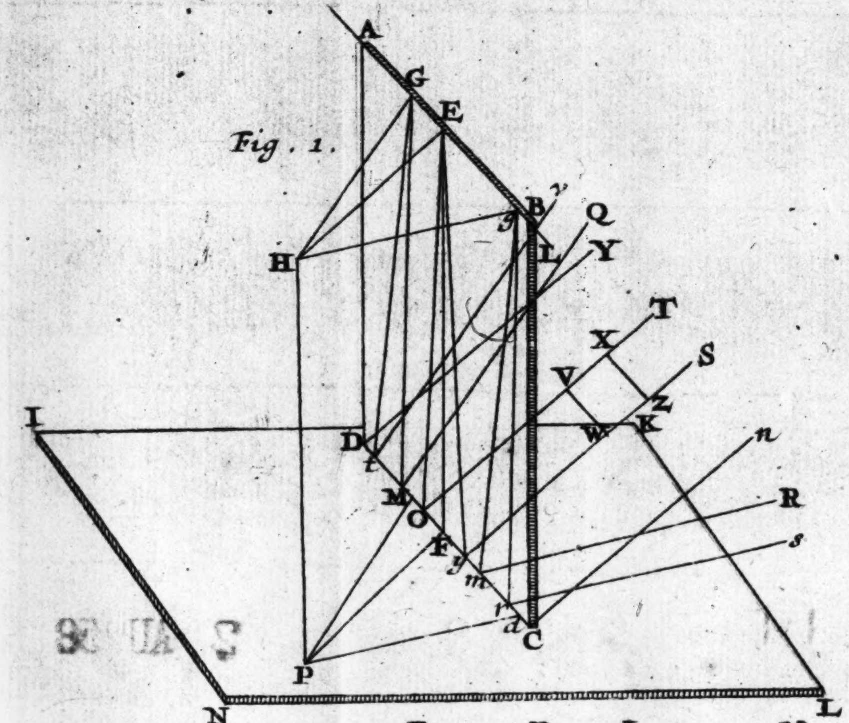
3. If the Curve MN were any other besides a Circle; its Property being given; by the *Methods of Tangents*, we can draw a Line, as AN *perpendicular* to the Curve, at the given Point of Incidence N; and then proceed as before. So that let the *Curvature*, be what it will, the Concourses of the Rayes as Refracted thereby, may be Practically and yet Geometrically trac'd out.

4. This Construction may easily be apply'd to any Sort of *Reflecting* Surface, as well as to *Refracting* ones; the Quantities *m*, *n*, which before were in the Ratio of the Refraction, being now put equal one to another.

FINIS.

Plate 1

Fig. 1.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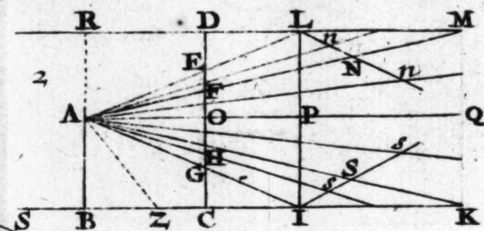
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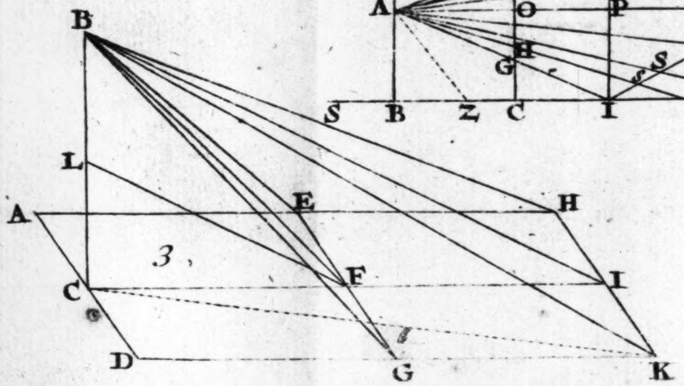
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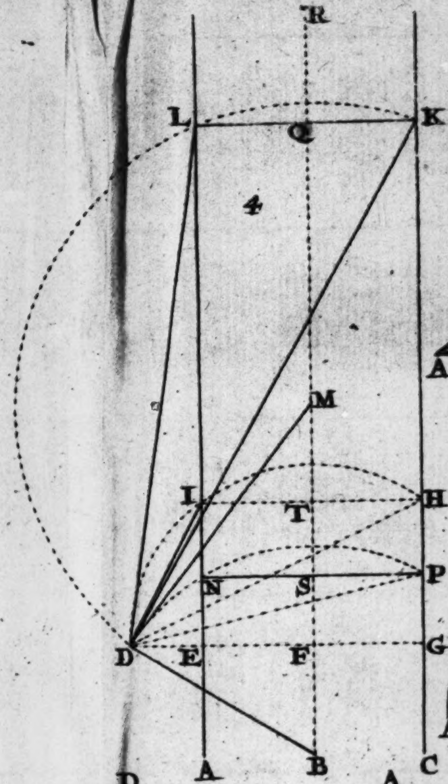
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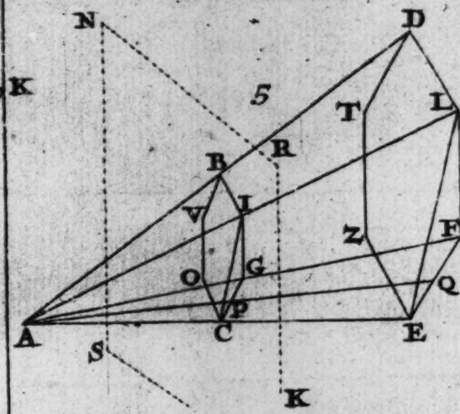
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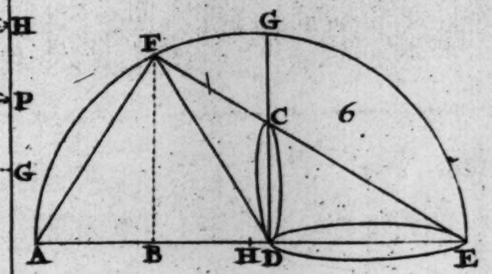
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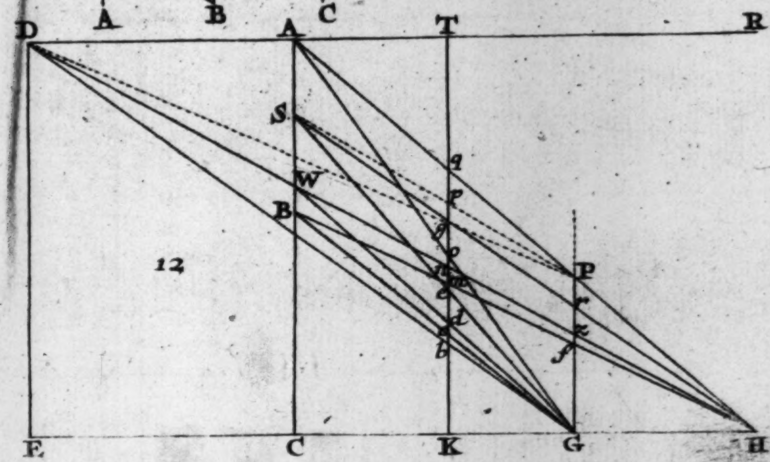
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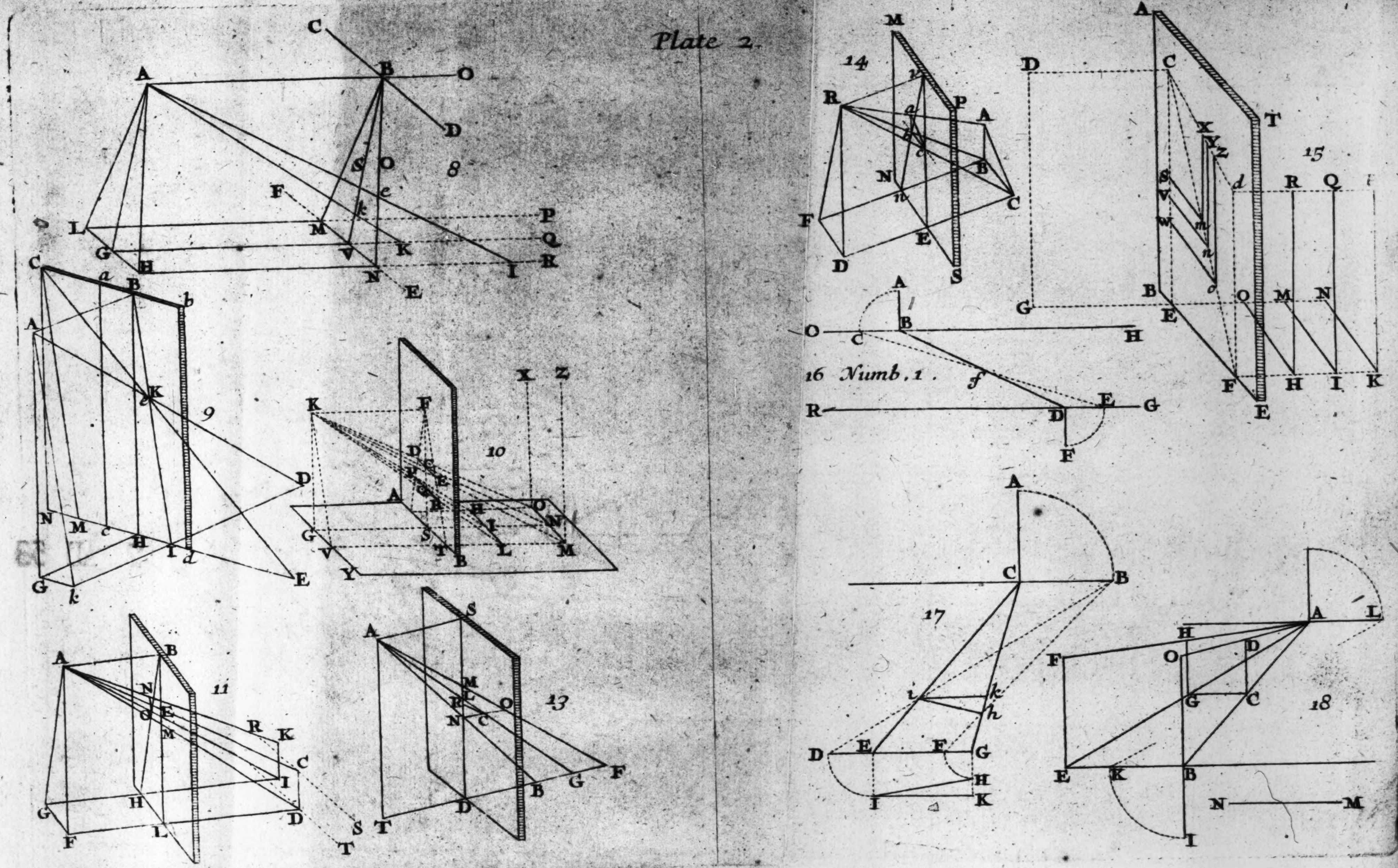
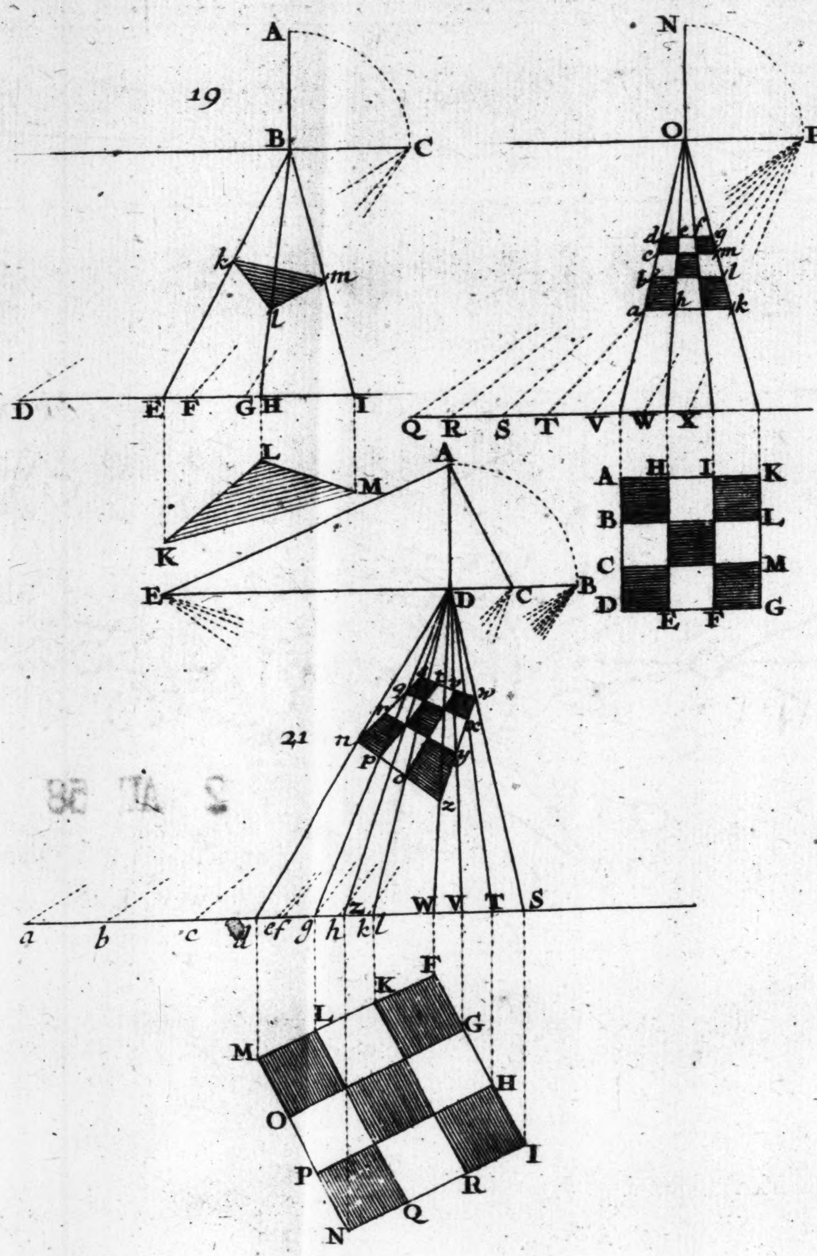


Plate 3



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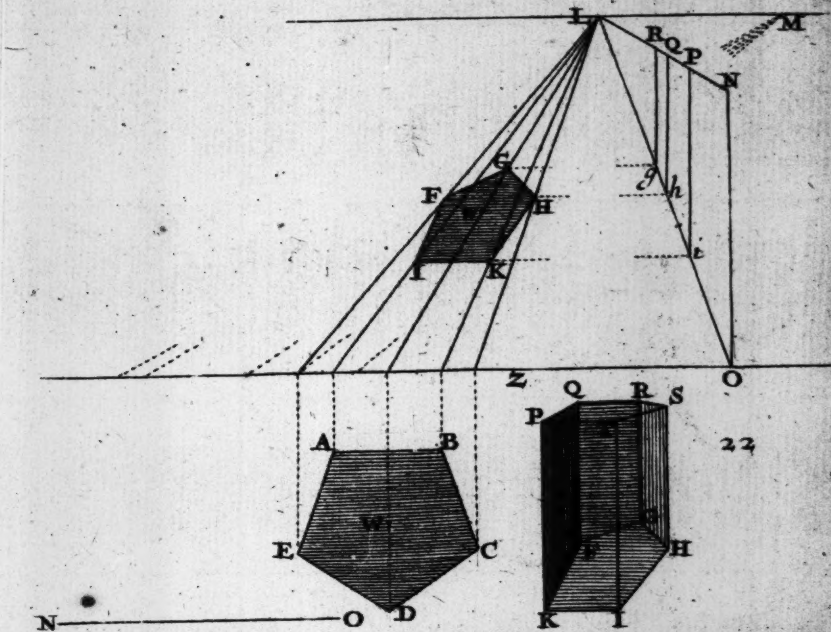

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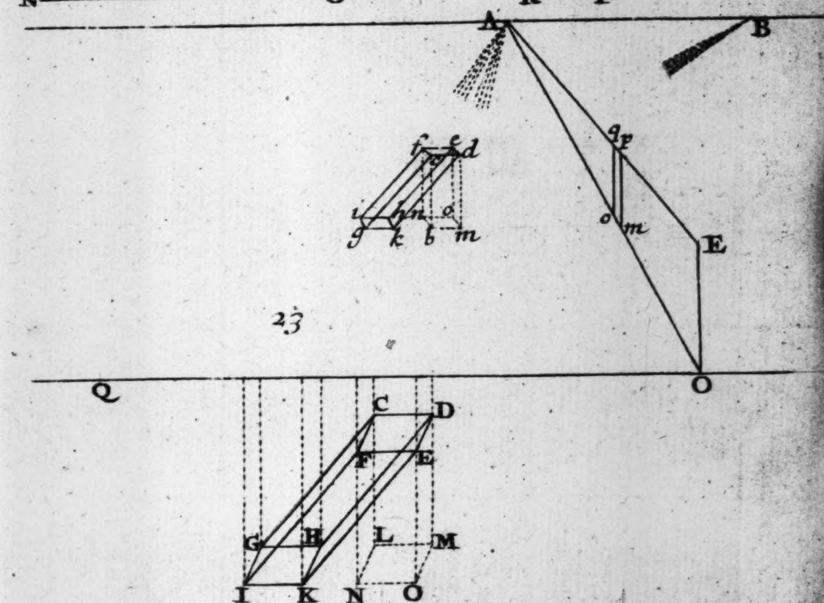

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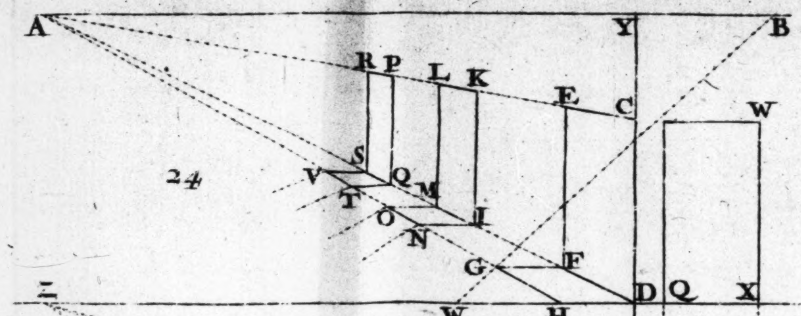
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B
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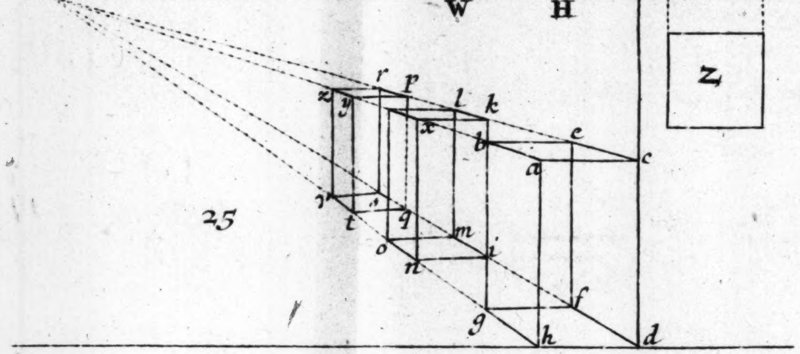
E
C
W
24
V S Q M
T O I
N
G F
Z
W H D Q X
Z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b c
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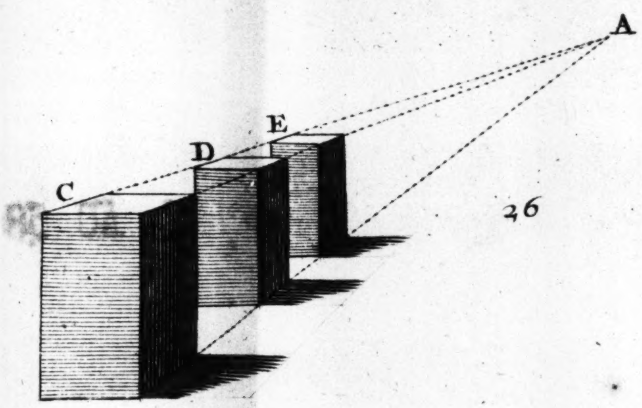
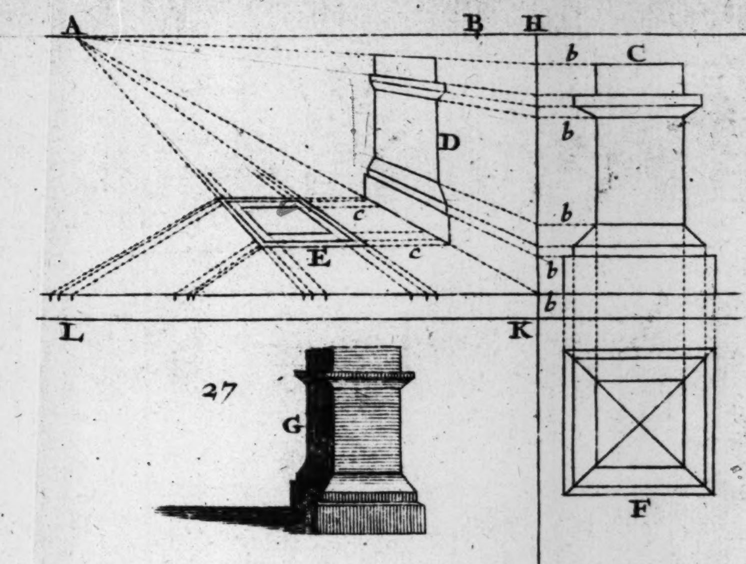
g h d


A
E
D
C
26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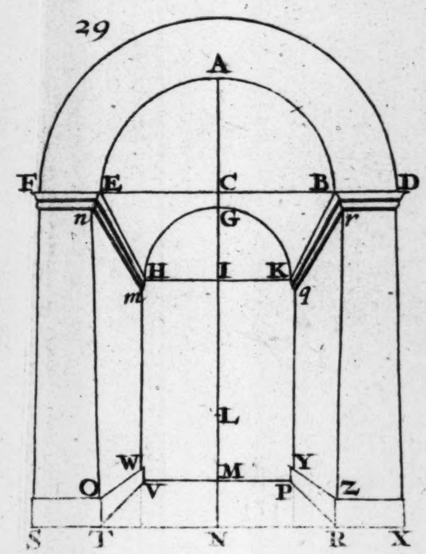
B H
b C
D
b
b
c
E c
b
b
b
L K
27
G
F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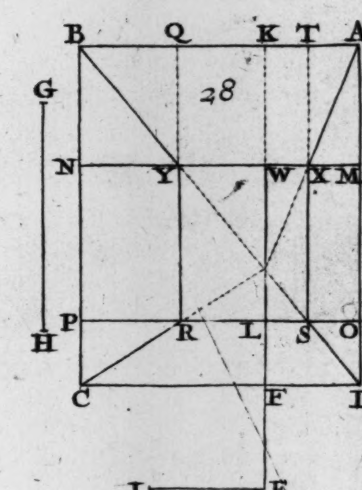
A
F E C B D
n G r
H I K
m q
L
W M Y
O V P Z
S T N R X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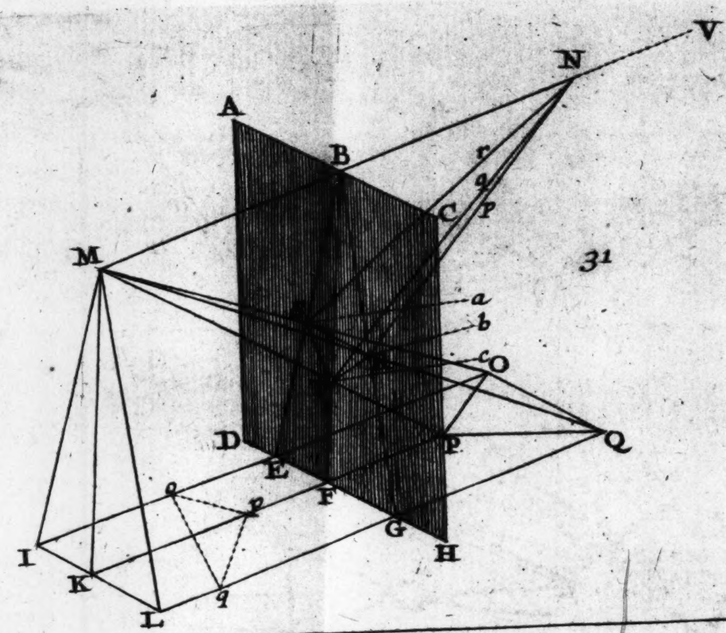
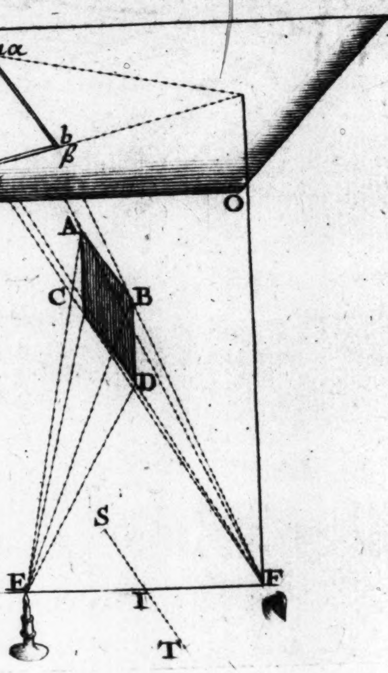
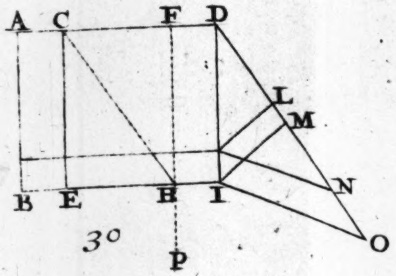
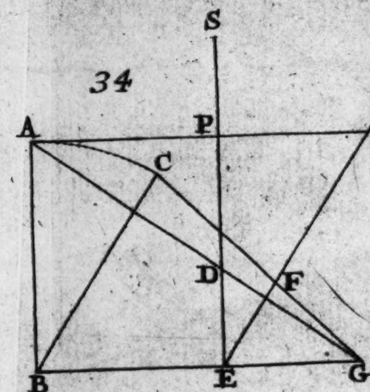
N Y W X M
P R L S O
H
C F D
I E

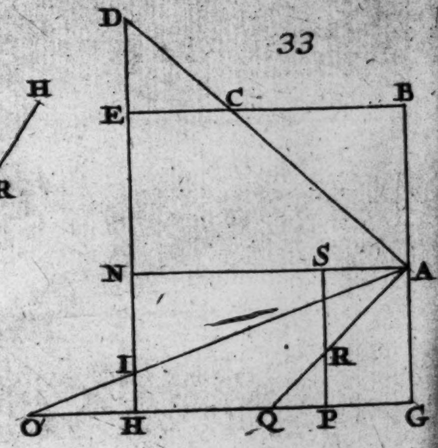
Plate 5.











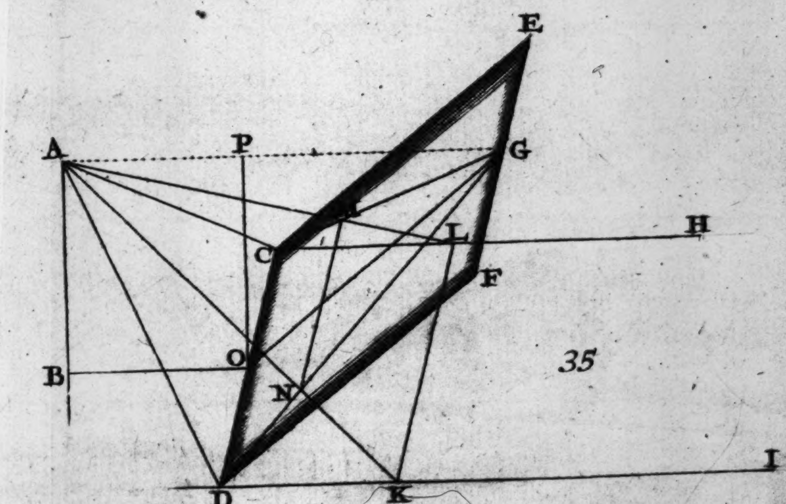


Plate 6.

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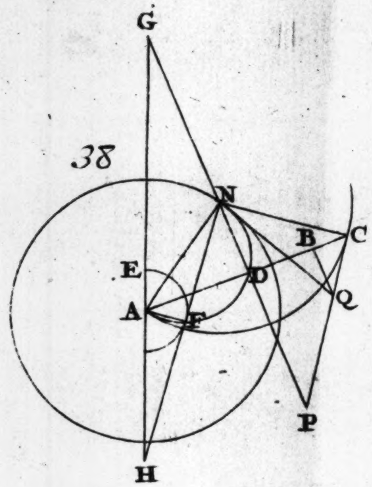

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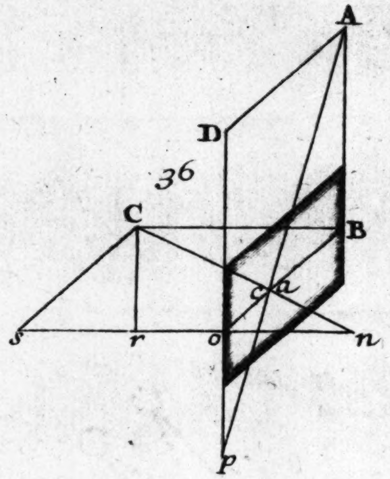

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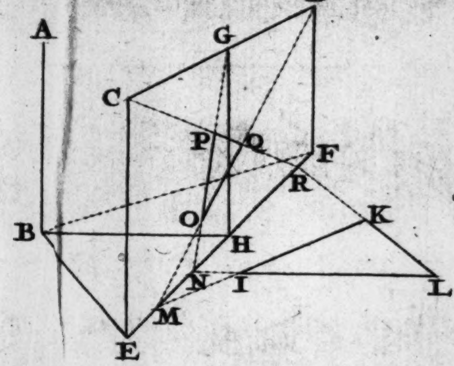

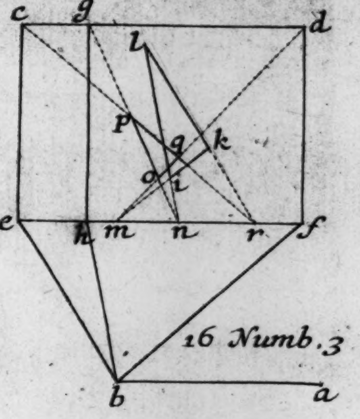

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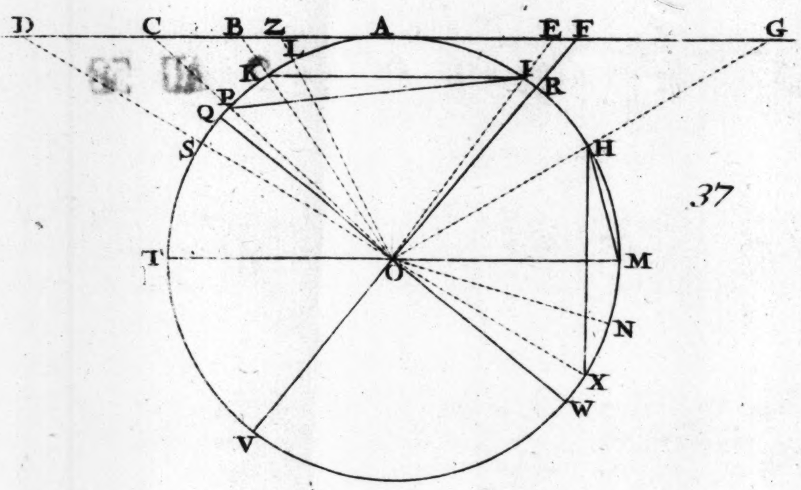

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